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POSITRON SURVIVAL IN TYPE II SUPERNOVAE

by

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Abstract

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In this work I investigate the possibility of Type II supernovae being the origin for positrons producing observed annihilation radiation observed toward the Galactic center. It was my contention that the decay of ^{56}Co coupled with falling densities would allow for the production and extended existence of positrons in the supernova outflow. Supernova 1987A has prompted many people to construct models of supernova outflow. I use the results of two existing models as the initial conditions in my models. I have created both an analytic and a computer model for the survival of positrons. These models show that while Type II supernovae fall short of the needed production of surviving positrons, the lower densities existing in Type I supernovae may be a more promising source.

Acknowledgements

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I. Introduction

Since the discovery of Supernova 1987A (a Type II supernova) in February of 1987 there has been a flurry of activity in the field of supernova modelling. The fact that the supernova occurred in the nearby Large Magellanic Cloud and that the progenitor star has almost universally been identified as Sk -69 202 (Woosley 1988; Woosley (in press); Nomoto et al. 1988; Shigeyama, Nomoto, and Hashimoto 1988) has allowed great progress to be made in this area. Prior to exploding, Sk -69 202 was seen as a B3-I blue supergiant with a surface temperature of approximately 16,000 K and a bolometric magnitude of -7.5 to -8.2. A blue supergiant becoming a Type II supernova shocked many people. It has been suggested that Sk -69 202 was once a red supergiant but it evolved back to the blue. Models have used low stellar metallicity and mass loss to get such a star to evolve back before exploding (Woosley 1988; Shigeyama, Nomoto, and Hashimoto 1988; Nomoto et al. 1988). As a result of the proximity of SN 1987A, there is a wealth of data on the light curve. Details of the middle light curve have put limits on the amount of radioactive ^{56}Ni (and thus the ^{56}Co) that was produced in the explosive silicon burning that took place as the shock propagated outward. Details of the middle light curve together with the early emergence of x- and γ-rays indicate that this nickel was mixed radially outward as well (Woosley 1988; Woosley (in press); Pinto and Woosley 1988; Shigeyama, Nomoto, and Hashimoto 1988; Nomoto et al. 1988). This radial mixing was driven by Rayleigh-Taylor instabilities caused by the deposition of γ-ray energy in the region of nickel production.

These new models allow many calculations to be made on the effects of supernovae on their galactic neighborhood. One of these will be investigated in this thesis; the possible contribution of positrons from a Type II supernova to the observed Galactic 511 keV γ-ray line. Observations of celestial annihilation radiation was pioneered by Haymes

et al. at Rice University in the late 60's and early 70's. They found a source of possible annihilation radiation towards the Galactic center. However, the source of the observed line was never pinpointed as positron annihilation because of inadequate energy resolution. The measured flux was $1.8 \times 10^{-3}\ \gamma\ cm^{-2}\ s^{-1}$ (if interpretted as a point source) when using a detector with an aperture of 24° FWHM in 1970 and 1971. Later, in 1974, a detector with an aperture of 15° FWHM was used and a flux of $0.80 \times 10^{-3}\ \gamma\ cm^{-2}\ s^{-1}$ was observed. Thus, evidence for a variable or diffuse source was seen from the beginning. Several groups have continued this work. In 1977 Bell/Sandia Labs were able to determine that the line was due to annihilation of positrons because of their improved energy resolution. They observed a flux of $1.22 \times 10^{-3}\ \gamma\ cm^{-2}\ s^{-1}$ towards the Galactic center with a 15° FWHM aperture. Also in 1977 the Centre d'Etude Spatiale des Rayonnements measured a flux of 4.2 or 3.2 $\times 10^{-3}\ \gamma\ cm^{-2}\ s^{-1}$ (depending on reduction method) using an aperture of 50° FWHM. Comparing their observations with earlier ones, they concluded that the observed flux depended on the instrument aperture. This high flux was confirmed by a group at the University of New Hampshire with a detector with an aperture of 100° FWHM in 1977. Riegler et al. made observations using the HEAO-3 satellite (35° FWHM) in 1979 and 1980. They found fluxes of 1.85 and 0.65 $\times 10^{-3}\ \gamma\ cm^{-2}\ s^{-1}$ respectively (Share et al. 1988). Ramaty and Lingenfelter saw this as evidence of a variable compact source that turned on in 1977 and off in 1979 plus a diffuse source (Ramaty and Lingenfelter 1987). Share et al. made observations using the 130° FWHM γ-ray spectrometer on the Solar Maximum Mission satellite from October to February yearly from 1980-81 to 1985-86. They report a flux of about $2.1 \times 10^{-3}\ \gamma\ cm^{-2}\ s^{-1}$ with less than a 30% yearly variation. This was significantly larger than observations made with smaller apertures within that same time period (see figure 1). Thus Share et al. conclude that the best explanation is a diffuse source of annihilation radiation and the case for a variable source was weakened because of the short period for variability needed (Share et al. 1988).

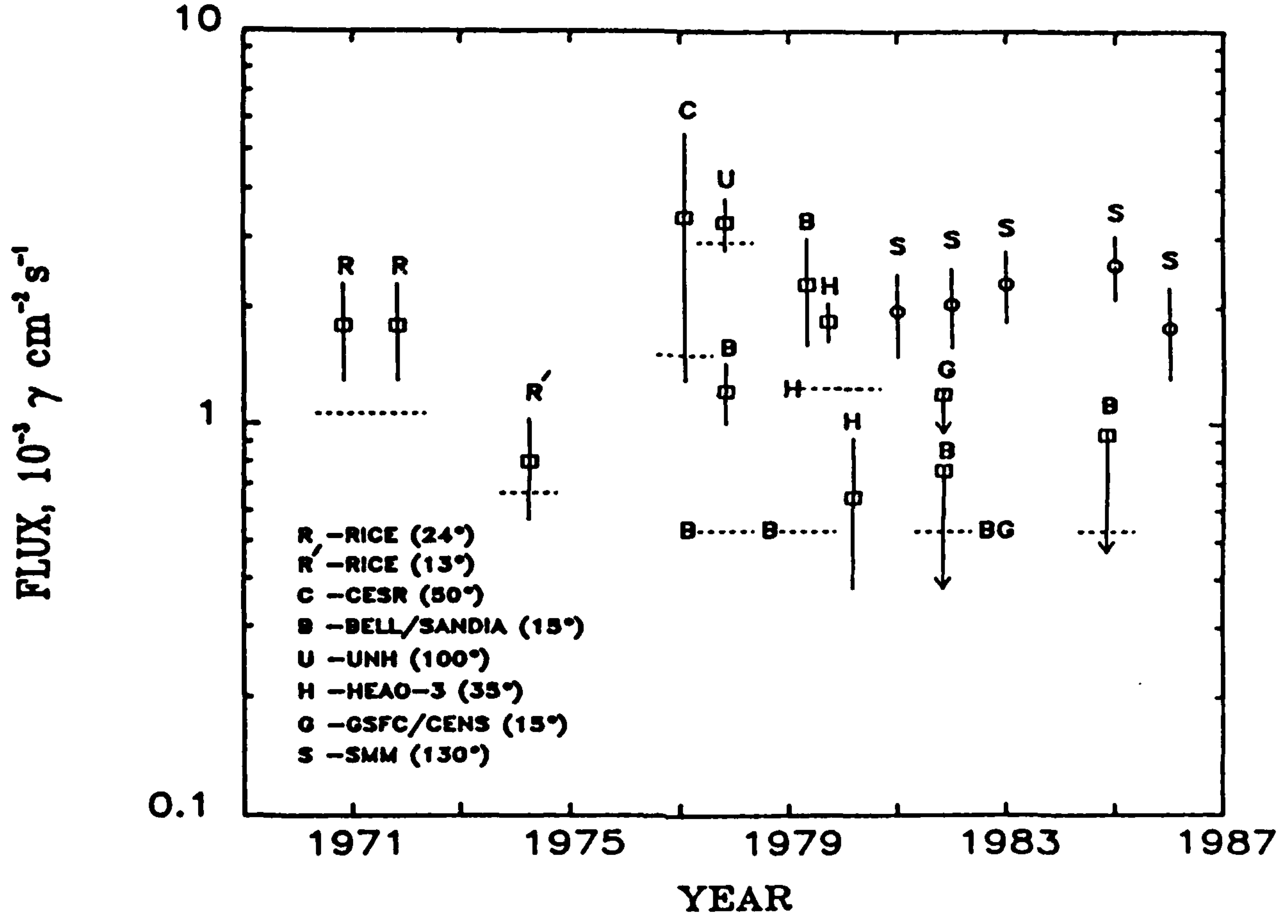


Figure 1: History of Galactic center 511 keV measurements. Note the general trend of larger apertures obtaining larger fluxes and the suggestion of a variable source turning on in 1977 and off in 1979 (Share et al. 1988).

Clayton (1973) suggested that the source of these positrons should be either ^{56}Co (daughter of ^{56}Ni) or ^{44}Ti. He predicted that both of these nuclei should be produced during explosive nucleosynthesis. The production of this nickel has been proven by the light curve of SN 1987A. He concluded that both were potentially adequate to account for the observed feature.

In this thesis I discuss the escape of positrons produced by the decay of cobalt to iron in a Type II supernova. I decided not to look at whether positrons could be "shot" out of a supernova, but whether a positron could survive for an extended period of time within the expanding gas. My motivations for this were two-fold:

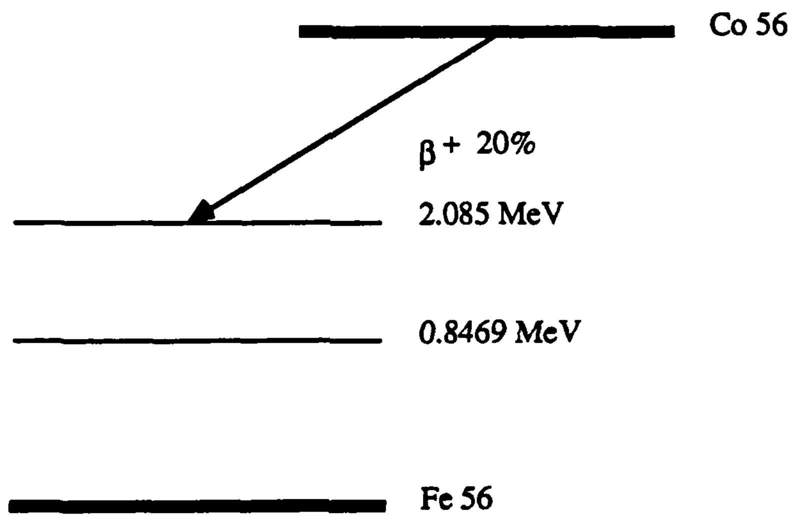
1) The uncertain structure of the magnetic fields within the supernova made it uncertain that a charged particle had any chance to escape.
2) The velocity profile of an expanding supernova provided an environment where the density of electrons would decrease rapidly (section IV).

The uncertain nature of the magnetic fields made them hard to incorperate into my model. The condition that the positrons were required to remain within the shell in which they were produced implied the existence of these fields.

The organization of this thesis is as follows. In sections II-V, I discuss the processes involved in the production, thermalization and annihilation of positrons. In section VI, I discuss published models of SN 1987A from which I determined the initial conditions for my models. Sections VII and VIII contain these models of positron survival. There is both an analytic and a computer model.

II. The β^+ Decay of ^{56}Co to ^{56}Fe

The decay of cobalt to iron is an important process because the energy liberated powers a supernova during the middle part of its light curve. We are particularly interested in the emission of positrons from this reaction as a possible source for the diffuse component of the background 511 keV γ-rays. The cobalt is formed by the decay of ^{56}Ni. This isotope of nickel has a half-life of 6.1 days; therefore, it all has essentially turned to cobalt at times of interest to us. I have therefore assumed that ^{56}Co was the explosive nucleosynthesis product. The half-life of ^{56}Co is 78.8 days (see figure 2). A positron is emitted when the cobalt decays to an excited state of iron. This route is taken 20% of the time.



The Q-value for electron capture is 4.57 Mev (Lederer et al. 1967). From this, one can determine that the maximum kinetic energy of an emitted positron is 1.46 MeV. The positron energy distribution function is of the form (Whaling 1960) :

$$N = kpw(w_0 - w)^2$$

where k = a normalization constant,

p = momentum of the positron,

w = total energy of the positron = KE + mc^2,

w_0= maximum total energy of the positron = KE_{max} + mc^2.

Thus N = kpw[KE_{max} - KE $]^2$. This can be simplified so the only variable is the kinetic energy:

$$N = \frac{k}{c} KE^{\frac{1}{2}} \left[KE + 2mc^2 \right]^{\frac{1}{2}} \left[KE + mc^2 \right] \left[KE_{max} - KE \right]^2 . \qquad \text{(II-1)}$$

To numerically follow the thermalization of these positrons I will find it convenient to approximate the spectrum as a weighted sequence of monoenergetic emissions. Breaking the energy range (0-1.46 MeV) into ten intervals, I calculate the fraction of positrons emitted in each interval by finding the area under a plot of equation (II-1) (see figure 3) to be:

Table II-1: Fraction of Positrons in Kinetic Energy Interval

Interval (MeV)	Midpoint Energy of Interval (MeV)	Fraction of Total Positrons
0.000 - 0.146	0.073	0.061
0.146 - 0.292	0.219	0.117
0.293 - 0.438	0.365	0.152
0.439 - 0.584	0.511	0.165
0.585 - 0.730	0.657	0.158
0.731 - 0.876	0.803	0.142
0.877 - 1.022	0.949	0.104
1.023 - 1.168	1.095	0.066
1.169 - 1.314	1.241	0.030
1.315 - 1.460	1.387	0.005

This fraction will be used to weight quantities when in determining a mean value with respect to energy.

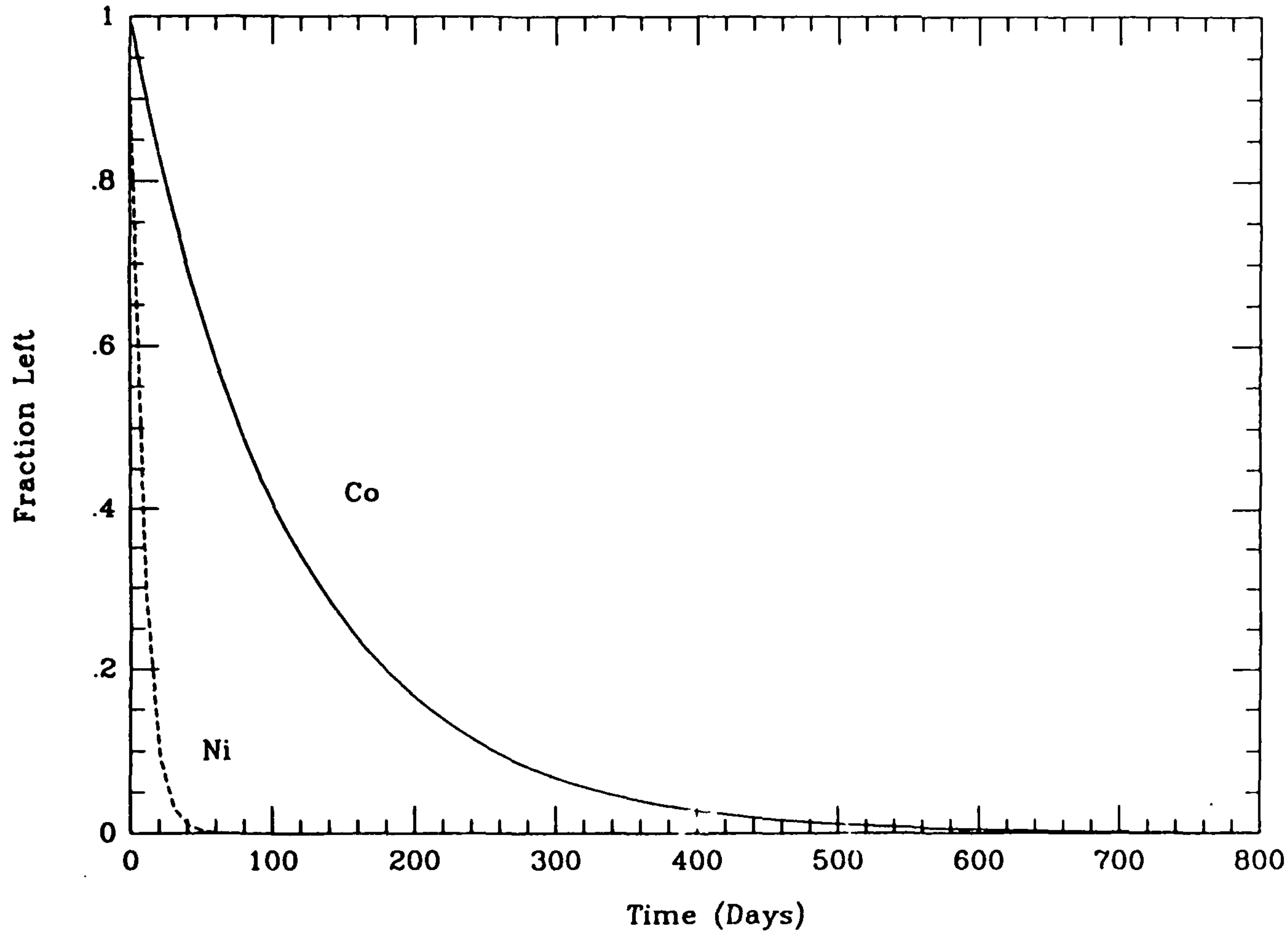


Figure 2: A comparison of the decay curves of ^{56}Ni and ^{56}Co.

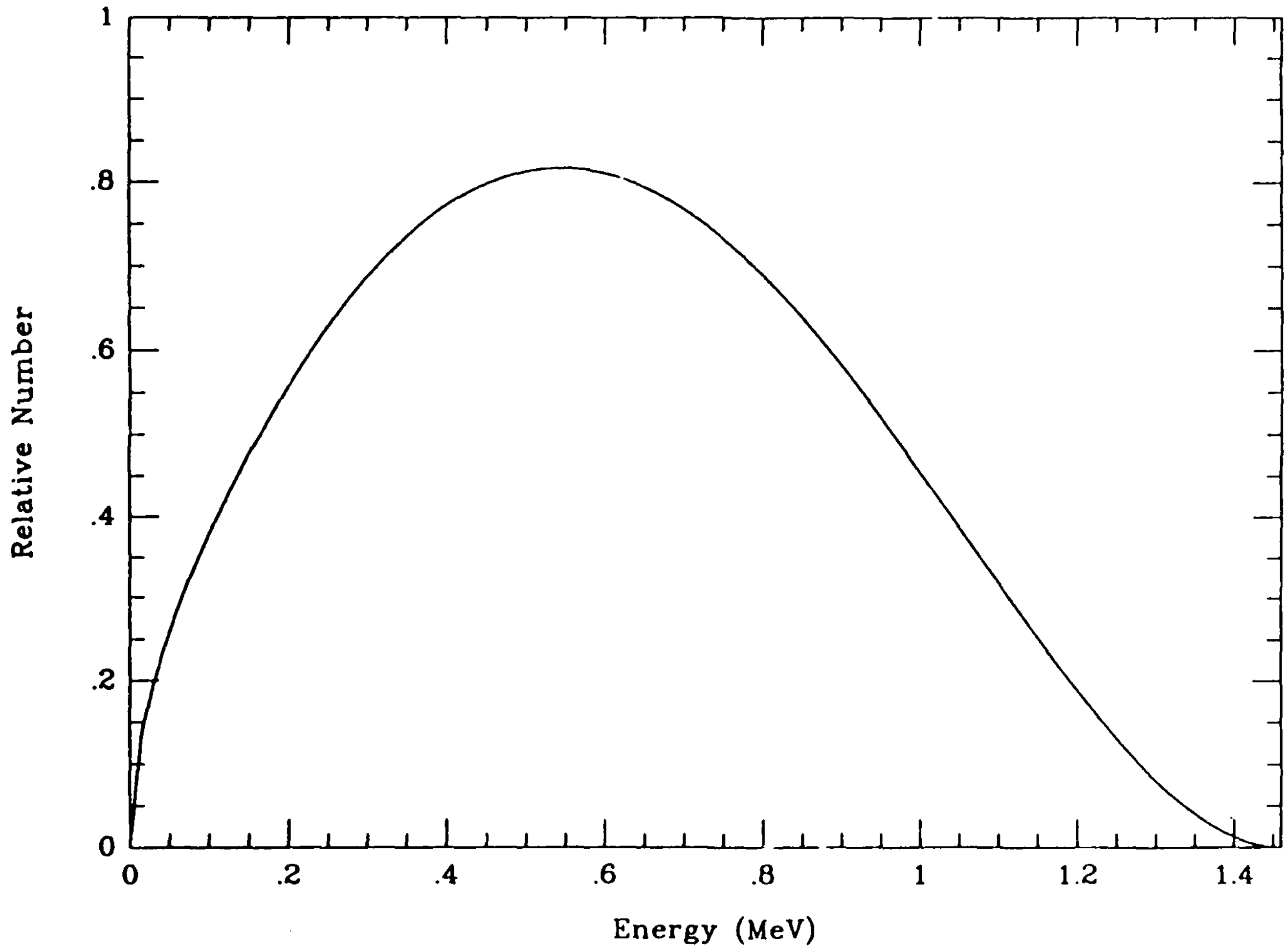


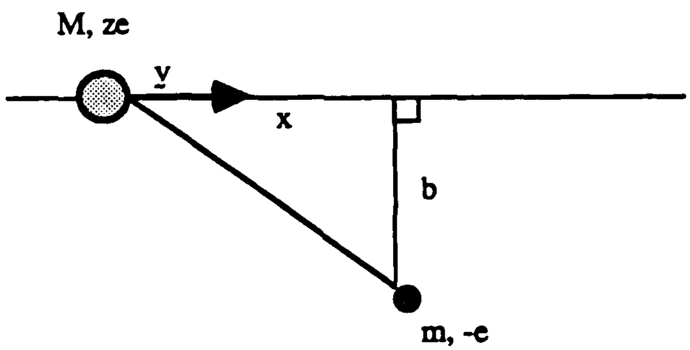
Figure 3: The energy distribution of positrons produced by the decay of ^{56}Co.

III. Positron Energy Loss in a Partially Ionized Medium

Once a positron is emitted, it loses energy in a variety of ways as it passes through matter. It will lose energy to the un-ionized component by inelastic collisions with the atomic electrons. In this process the electron will be removed from the atom or it will move to an excited state. The positrons will lose energy to the ionized component by interacting with the free electrons. The positrons will also radiate energy via Bremsstrahlung when interacting with nuclei. I derive expressions for these in order to calculate thermalization times for positrons emitted by ^{56}Co.

IIIa) Coulomb collisions with bound electrons

The energy loss rate for inelastic coulomb collisions with bound electrons has been calculated by many people. The general form of this loss rate can be gotten by a simple classical calculation (Whaling 1960). Consider an ion of mass M, charge ze, and velocity v. Let b be the impact parameter.



The momentum gained by the electron is:

$$\int_{-\infty}^{\infty} F\,dt \qquad \text{where } F = \frac{ze^2}{x^2 + b^2}.$$

If the electron is assumed to move very little during the encounter this can be simplified to:

$$\Delta p = \int_{-\infty}^{\infty} F_{\perp}\, dt \qquad \text{where } F_{\perp} = F \sin\Theta = \frac{ze^2 b}{\left(x^2 + b^2\right)^{\frac{3}{2}}} .$$

With the substitution $dx = v\, dt$,

$$\Delta p = \int_{-\infty}^{\infty} \frac{F_{\perp}\, dx}{v} = \frac{2ze^2}{bv} .$$

If the electron is regarded as free, then it acquires kinetic energy:

$$\Delta E = \frac{(\Delta p)^2}{2m}$$

The electron may be quasi-free if the Coulomb force from the passing ion greatly exceeds that holding the electron in its orbit. Therefore the amount of energy transferred from the ion to a bound electron is:

$$\Delta E \equiv \frac{2z^2e^4}{b^2v^2m} .$$

If there are NZ electrons per cubic centimeter, then the ion will encounter $2\pi b\, db\, dx$ NZ electrons with an impact parameter between b and b+db in traveling a distance dx. Thus the energy lost, dE, will be $2\pi b\, db\, dx$ NZ ΔE. The total amount of energy lost in traveling a distance dx is found by integrating over all impact parameters:

$$\frac{dE}{dx} = \frac{4\pi z^2e^4NZ}{mv^2} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{4\pi z^2e^4NZ}{mv^2} \ln\left(\frac{b_{max}}{b_{min}}\right),$$

if z=1 (as for positrons)

$$\frac{dE}{dx} = \left(\frac{4\pi e^4}{mc^2}\right)\left(\frac{NZ}{\beta^2}\right)\ln\left(\frac{b_{max}}{b_{min}}\right). \qquad \text{(III-1)}$$

The values of b_{min} and b_{max} are estimated in different ways. One example is:

$$b_{min} = \max\left[\frac{Ze^2}{\gamma m v^2}, \frac{h}{\gamma m v}\right] \text{ and } b_{max} = \frac{\gamma v}{\omega}.$$

where γ is the relativistic correction factor and ω is a characteristic atomic frequency of motion (Jackson 1975). Physically, here the minimum impact parameter is either the impact parameter where the maximum allowable energy ($2\gamma^2 m v^2$) is transferred to the bound electron using the above form for ΔE or the minimum extent of the electron's wave packet. The maximum impact parameter is gotten by setting the collision time equal to the orbital period. If the collision time is greater than the orbital period there is no net transfer of energy to the atomic electron.

More in depth calculations yield various results. Bethe derived a quantum mechanical formula for the energy loss of an electron owing to ionization of bound electrons as it passes through matter with relativistic corrections and corrections due to the electron spin (Heitler 1934; Wu 1960):

$$\frac{dE}{dx} = \frac{2\pi e^4 NZ_{bound}}{mv^2}\left[\ln\left(\frac{\gamma^2 m v^2 E}{2\bar{I}^2}\right) + \Delta\right] - \Gamma_{pol}, \qquad \text{(III-2)}$$

where I is the mean ionization potential ($\approx$11.5 eV x Z),

NZ_{bound} is the density of atomic electrons,

$\beta = v/c$,

$$\gamma = \left(\sqrt{1-\left(\frac{v}{c}\right)^2}\right)^{-1},$$

$$\Delta = \left(1-\beta^2\right) - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right)\ln 2 + \frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2,$$

and Γ_{pol} is a correction term resulting from dielectric properties of the medium.

Equation (III-2) can be transformed into:

$$\frac{dE}{dx} = \frac{4\pi e^4}{mc^2}\left(\frac{NZ_{bound}}{\beta^2}\right)\left\{\ln\left[\beta\left(\frac{E+mc^2}{\bar{I}}\right)\left(\frac{E}{mc^2}\right)^{\frac{1}{2}}\right] - \frac{1}{2}\ln 2 + \frac{1}{2}\Delta\right\} - \Gamma_{pol}. \quad \text{(III–3)}$$

This gives a factor multiplying the logarithm that is the same as in the classical equation (III-1). In order for equation (III-2) to apply to positrons I believe that the argument of the logarithm should be increased by a factor of two in equation (III-2). For the case of an incoming electron arguments were made that the maximum energy transfer would be 1/4 mv^2 instead of 1/2 mv^2 because it is impossible to distinguish whether the detected outgoing electron is the target or the projectile. But a positron and electron are distinguishable. This would eliminate the -1/2 ln 2 in equation (III-3). The details of the calculation were not provided making it impossible to tell for certain. Since it is not the purpose of this thesis to investigate details of energy loss, I will assume equation (III-3) is sufficiently correct for positrons.

Another version of this formula was listed for positrons or electrons by M. Zombeck in Special Report 386 of the Smithsonian Astrophysical Observatory (Sect. 13, p. 23, 1980):

$$\frac{dE}{dx} = 4\pi r_o^2 \frac{mc^2}{\beta^2} NZ_{bound}\left\{\ln\left[\beta\left(\frac{E+mc^2}{\bar{I}}\right)\left(\frac{E}{mc^2}\right)^{\frac{1}{2}}\right] - \frac{1}{2}\beta^2\right\}$$

$$= \frac{4\pi e^4}{mc^2}\left(\frac{NZ_{bound}}{\beta^2}\right)\left\{ \ln\left[\beta\left(\frac{E+mc^2}{I}\right)\left(\frac{E}{mc^2}\right)^{\frac{1}{2}}\right] - \frac{1}{2}\beta^2 \right\}, \qquad \text{(III-4)}$$

where $r_o = \frac{e^2}{mc^2}$ = the classical electron radius ,

and as before, I=the mean ionization potential, β=v/c, and NZ_{bound}=the density of atomic electrons.

A third version of this formula is given for electrons by J. Jackson (op cit pp. 619-637):

$$\frac{dE}{dx} = \frac{4\pi e^4}{mc^2}\left(\frac{NZ_{bound}}{\beta^2}\right) \ln\left[(\gamma - 1)\sqrt{\frac{\gamma+1}{2}}\,\frac{mc^2}{h\langle\omega\rangle}\right], \qquad \text{(III-5)}$$

where $I \approx h\langle\omega\rangle$.

These formulae differ only in the arguments of the logarithm and the correction terms involving β. A comparison of this correction term follows.

Table III-1: Comparison of the correction terms of equations (III-3) and (III-4)

β	β^2	$-\frac{1}{2}\beta$	$\frac{1}{2}\Delta$	$-\frac{1}{2}\ln 2 + \frac{1}{2}\Delta$
0.1	0.01	- 0.005	0.15	- 0.20
0.2	0.04	- 0.020	0.13	- 0.21
0.3	0.09	- 0.045	0.11	- 0.24
0.4	0.16	- 0.080	0.08	- 0.27
0.5	0.25	- 0.125	0.04	- 0.31
0.6	0.36	- 0.180	- 0.01	- 0.36
0.7	0.49	- 0.245	- 0.06	- 0.41
0.8	0.64	- 0.320	- 0.10	- 0.45
0.9	0.81	- 0.405	- 0.12	- 0.47

Since the value of the logarithm term was of order 10 for the initial energies of the

positrons, I considered the differences in these small terms unimportant.

IIb) Plasma excitation

The positrons also interact with the free electrons in a medium. Bussard claims that positrons lose energy to the ionized component of a medium by exciting plasma waves (Bussard, Ramaty, and Drachamn 1979). The formula that he quotes from Book and Ali (1975) is :

$$\frac{dE}{dx} = 1.3\times10^{-13}\,\frac{n_e}{E}\left[M\left(\frac{E}{kT}\right) - M'\left(\frac{E}{kT}\right)\right]\ln\Lambda \quad \text{eV/cm} \qquad \text{(III-6)}$$

where $M = \frac{2}{\sqrt{\pi}}\int_0^{\frac{E}{kT}} dx\,\sqrt{x}\; e^{-x}$,

n_e= the free electron density = NZ_{free},

E = the positron energy in eV,

$$\Lambda = \left(\frac{kT}{4\pi n_e e^2}\right)^{\frac{1}{2}}\left[\max\left(\frac{2e^2}{m_e u^2}, \frac{h}{m_e u}\right)\right]^{-1} ,$$

$$u = \left(\frac{2E}{m_e}\right)^{\frac{1}{2}} - \left(\frac{8kT}{\pi m_e}\right)^{\frac{1}{2}}.$$

Bussard et al. went into no details about the origin of this equation. Jackson deals with the problem of an ion of charge ze passing through an electronic plasma (op cit pp. 641-643). The interactions are broken into two types depending on whether the impact parameter is larger than or smaller than a Debye length. When the impact parameter is smaller than a Debye length there is a two-body screened potential interaction. When the impact parameter is larger than a Debye length there is a collective response of the medium and plasma oscillations are excited where the energy of these oscillations is extracted from the particle. Thus Jackson's formula is broken into two parts:

$$\frac{dE}{dx} = \frac{(ze)^2}{v^2}\omega_p^2 \ln\left(\frac{1.123k_D v}{\omega_p}\right) \quad \text{for} \quad b > \frac{1}{k_D} \quad ,$$

$$\frac{dE}{dx} = \frac{(ze)^2}{v^2}\omega_p^2 \ln\left(\frac{1}{1.47k_D b_{min}}\right) \quad \text{for} \quad b < \frac{1}{k_D} \ ,$$

$$\text{where} \ \omega_p^2 = \frac{4\pi n_e e^2}{m} \quad , \ k_D^2 = \frac{4\pi n_e e^2}{kT} \quad , \quad \text{and} \ b_{min} = \max\left(\frac{ze^2}{\gamma m v^2}, \frac{h}{\gamma m v}\right) .$$

After making the appropriate substitutions:

$$\frac{dE}{dx} = \frac{4\pi e^4}{mc^2}\left(\frac{n_e}{\beta^2}\right)\ln\left[1.123\left(\frac{mv^2}{kT}\right)^{\frac{1}{2}}\right] \quad \text{for } b > \frac{1}{k_D} \ , \qquad \text{(III-7)}$$

$$\frac{dE}{dx} = \frac{4\pi e^4}{mc^2}\left(\frac{n_e}{\beta^2}\right)\ln\left\{\left(\frac{kT}{4\pi n_e e^2}\right)^{\frac{1}{2}}\left[1.47 \max\left(\frac{e^2}{\gamma m v^2}, \frac{h}{\gamma m v}\right)\right]^{-1}\right\} \ \text{for } b < \frac{1}{k_D}, \quad \text{(III-8)}$$

where $\frac{4\pi e^4}{mc^2} = 5.1\text{x}10^{-25}$ MeV-cm^2 ,

and as before n_e= the free electron density = NZ_{free}.

In order to make these formulae appropriate for positrons I believe that the form of b_{min} should be altered to that of an electron:

$$b_{min} = \max\left(\frac{e^2}{\gamma m v^2}, \frac{h}{mc}\sqrt{\frac{2}{\gamma - 1}}\right) .$$

Jackson states that his calculations are for non-relativistic particles. Bussard makes no

statement about the energies where his formula is valid. Jackson's small impact parameter formula and Bussard's formula are nearly equal if one assumes that the latter is also non-relativistic. But the problem is that they seem to be explaining two different types of interactions, plasma oscillations and two-particle interactions. This is one of my major dilemmas, considering that plasma energy loss is not the objective of this thesis. What follows is a comparison of the coefficients of Bussard's and Jackson's formulae (assuming both formulae are valid at the listed energies).

Table III-2: Comparison of the coefficients in equations (III-6) and (III-7)

E (MeV)	β^2	$\frac{1.3\text{x}10^{-25}}{E}$ MeV-cm^2	$\frac{5.1\text{x}10^{-25}}{\beta^2}$ Mev-cm^2
1.50	0.94	0.87	5.5
1.00	0.89	1.3	5.8
0.80	0.85	1.6	6.0
0.60	0.79	2.2	6.5
0.40	0.69	3.3	7.4
0.20	0.48	6.5	11
0.10	0.30	13	17
0.05	0.17	26	30
0.01	0.04	130	130

(all values are x10^{-25})

The total energy loss due to the ionized component is either equation (III-6):

$$\frac{dE}{dx} = 1.3\text{x}10^{-25}\left(\frac{NZ_{free}}{E}\right)(M - M')\ \ln\left[\left(\frac{kT}{4\pi n_e e^2}\right)^{\frac{1}{2}}\left[\max\left(\frac{2e^2}{mu^2};\frac{h}{mu}\right)\right]^{-1}\right] \text{MeV/cm}$$

(E in MeV)

or by adding equations (III-7) and (III-8):

$$\frac{dE}{dx} = 5.1\text{x}10^{-25}\left(\frac{NZ_{free}}{\beta^2}\right)\ln\left[0.764\left(\frac{mv^2}{4\pi n_e e^2}\right)^{\frac{1}{2}}\left[\max\left(\frac{e^2}{\gamma m v^2}, \frac{h}{mc}\sqrt{\frac{2}{\gamma - 1}}\right)\right]^{-1}\right]. \quad \text{(III-9)}$$

(in MeV / cm)

IIc) Radiative losses due to Bremsstrahlung

The radiative loss rate due to Bremsstrahlung has been calculated by several people also. Bethe and Heitler calculated the loss rate to be (Wu 1960):

$$\frac{dE}{dx} = NE_0\Phi \quad ; \quad \Phi = \frac{Z(Z+1)}{137}\left(\frac{e^2}{mc^2}\right)^2\left[4\ln\left(\frac{2E_0}{mc^2}\right) - \frac{4}{3}\right] \qquad \text{(III-10)}$$

where $mc^2 << E_0 << \frac{137\ mc^2}{Z^{\frac{1}{2}}}$ and screening of the nucleus is neglected.

The ratio of radiative energy loss to ionization loss is given approximately by:

$$\frac{(dE)_{RAD}}{(dE)_{ION}} \approx \frac{E\,Z}{1600\ mc^2} = 0.0685\text{x}E\ (\text{MeV}). \qquad \text{(III-11)}$$

Thus this formula predicts radiative losses to be small at our energies.

Jackson also has words of wisdom on this subject (op cit pp. 708-719). He gives energy loss equations for the nonrelativistic limit and the ultrarelativistic limit. The ratios of radiative losses to ionization losses are:

$$\frac{(dE)_{RAD}}{(dE)_{ION}} = \frac{4}{3\pi}\left(\frac{Z}{137}\right)\frac{m}{M}\beta^2\ \frac{1}{\ln B} \qquad \text{(nonrelativistic)} \qquad \text{(III-12)}$$

and

$$\frac{(dE)_{RAD}}{(dE)_{ION}} = \frac{4}{3\pi}\left(\frac{Z}{137}\right)\frac{m}{M}\left[\frac{\ln\left(\frac{233\,M}{Z^{\frac{1}{3}}m}\right)}{\ln B}\right]\gamma \quad (\text{ ultrarelativistic, } \gamma >> 1 \text{ }) \qquad (\text{III-13})$$

where B is the argument of the logarithm in the ionization energy loss equation.

These formulae yield ratios of a few ten thousandth's of a percent and a few percent respectively. Zombeck also has a formula for radiative losses (op cit Sect. 13 , p. 23):

$$\frac{dE}{dx} = 3.09\text{x}10^{-27}\,NZ^2\,(\,E + mc^2\,) \quad \text{Mev/cm} \;. \qquad (\text{III-14})$$

This is also negligibly small compared to the ionization loss rate for our case. Thus I feel that Bremsstrahlung can be safely ignored in our calculations. I believe that Cherenkov can also be ignored because of the small rate of loss (Wu 1960), if it exists at all.

IIId) Conclusion

From these results I must choose expressions for the energy loss of ^{56}Co positrons in the supernova interior. Because the fraction of positrons that survive the expansion is only weakly sensitive to the differences in thermalization times the choice of which to use is not crucial. I chose to use equation (III-4) and equation (III-9) (see figures 4 and 5).

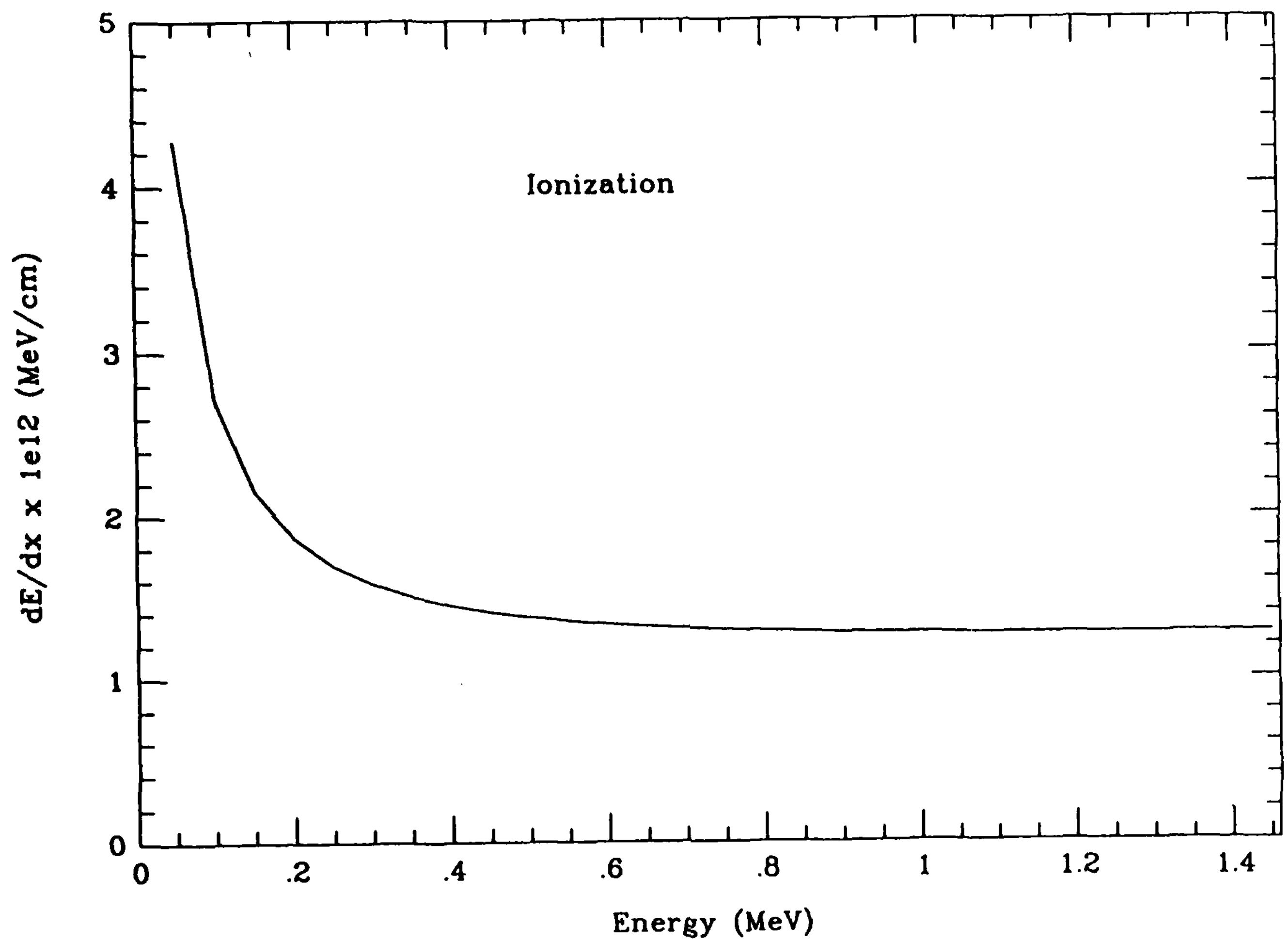


Figure 4: A plot of the ionization loss (equation (III-4)) with the number of bound electrons per atom set at 24.5 and the density of atoms set at 1.08×10^{10} atoms/cm^3. Note the large increase in dE/dx as the energy falls below 0.2 MeV.

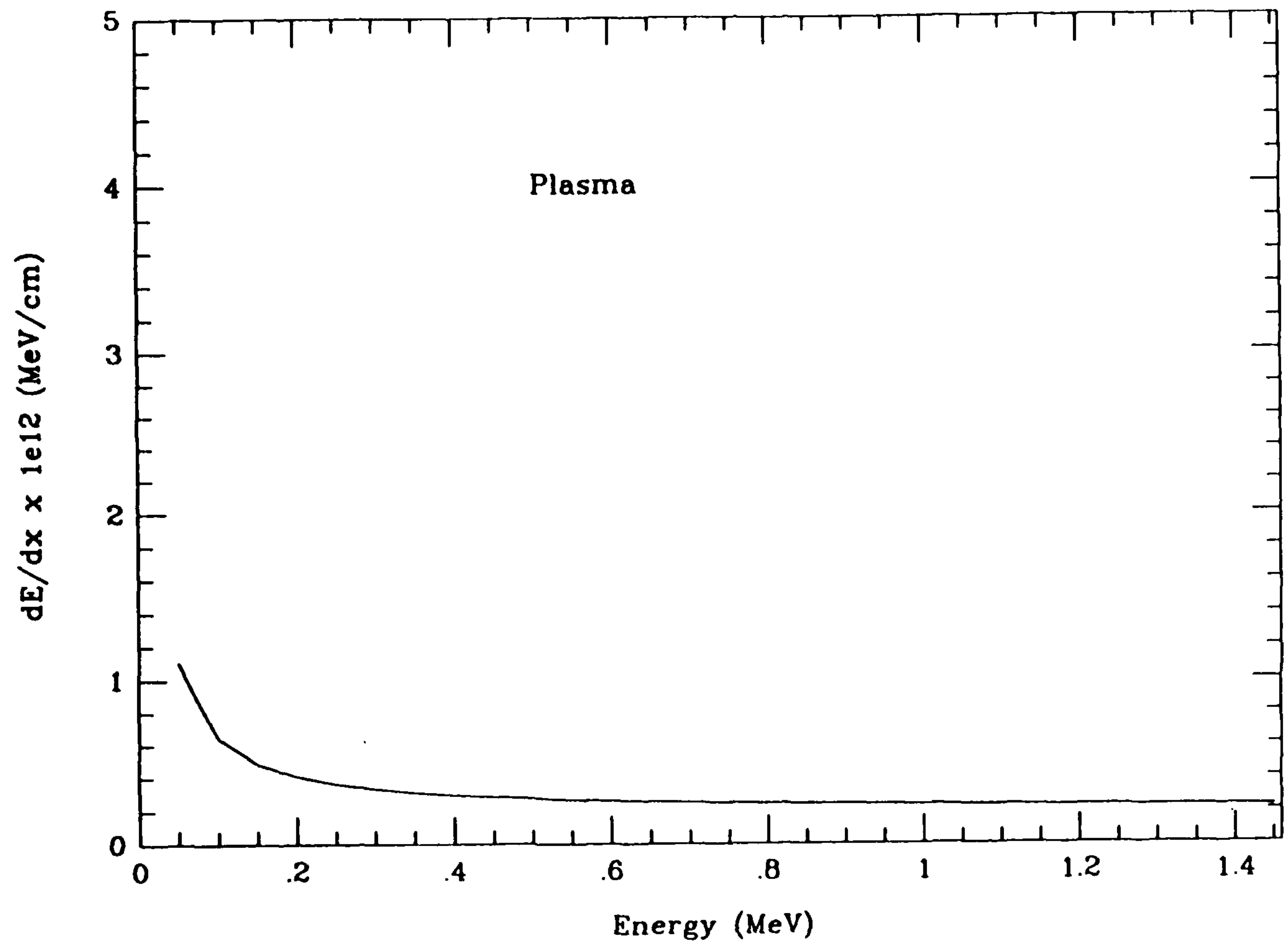
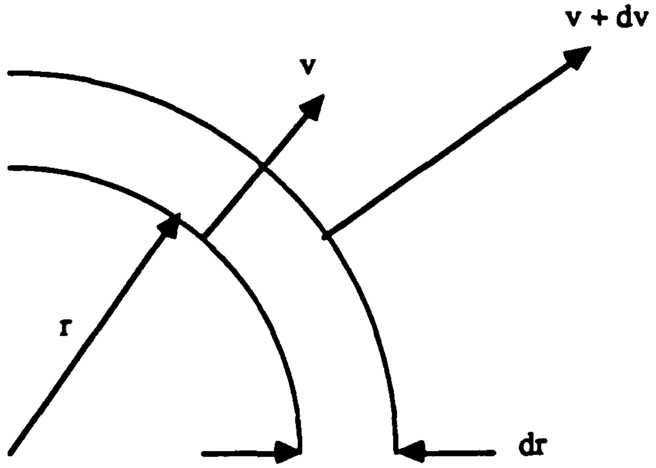


Figure 5: A plot of the energy loss due to plasma excitation (equation (III-9)). The number of free electrons per atom was set at 1.5 and the density of atoms was set at 1.08×10^{10} atoms/cm^3.

IV. Hubble Flow and Its Impact on the Density Evolution of Supernovae

As I have shown, the energy loss rate for a positron passing through matter depends on the density of that matter. The rate of positron annihilation also depends on the density of the matter, specifically the electron density. Thus I had to find an expression for the density evolution of the supernova ejecta. An important step in simplifying the density evolution of a spherical shell in a supernova was the assumption that the expanding gas is in a Hubble-type flow. Hubble flow simply means that the velocity at a given point in the supernova is proportional to its distance from the center of the explosion. It is easy to understand why this type of outward flow should occur. As the shock propagates outward each shell is pushing on the layers on top of it. The outer shells are pushing on less matter than inner ones and therefore should accelerate to a larger velocity. This becomes true in supernovae after acceleration has ended and after the size is much greater than the initial size. From this time on the ratio of velocity, $v(r)$, to distance from the center, r, is a constant for all r. This also means that there is a velocity gradient across a spherical shell. If one considers such a shell, it can be shown as follows that the density within that shell decreases as r^{-3}.



As $t \Rightarrow t + dt$: $\quad r \Rightarrow r + vdt$

$$r + dr \Rightarrow r + dr + (v + dv)dt.$$

From this, one can show that the thickness of the shell has increased to $dr + dvdt = dr(1+ (V_R/R)dt)$ where V_R is the velocity of the outermost region of the supernova and R is the radius of that region. It is easy to show $dv = dr\ (V_R/R)$ because in a Hubble flow $v(r)/r = V_R/R$ = constant. To show that the density in this region is proportional to t^{-3} one needs to use conservation of mass within the shell:

$$M(t) = M(t + dt),$$

where $\quad M(t) = 4\pi r(t)^2\, dr(t)\, \rho(t)$, and

$$M(t + dt) = 4\pi r(t + dt)^2\, dr(t + dt)\, \rho(t + dt).$$

Setting $M(t) = M(t + dt)$:

$$\Longrightarrow \quad 4\pi\, r(t)^2\, dr(t)\, \rho(t) = 4\pi\, r(t + dt)^2\, dr(t + dt)\, \rho(t + dt),$$

Keeping terms only to first order in dt:

$$4\pi r(t)^2\, dr(t)\, \rho(t) = 4\pi\, [r(t)^2 + 2r(t)v\, dt]\, dr(t)\, [1 + (V_R/R)\, dt]\, \rho(t + dt),$$

$$\Longrightarrow \quad r(t)^2\, \rho(t) = [r(t)^2 + 2r(t)v\, dt]\, [1 + (V_R/R)\, dt]\, \rho(t + dt),$$

$$\Longrightarrow \quad 0 = r^2\, d\rho + [2rv + r^2(V_R/R)]\, \rho\, dt\,,$$

$$= r^2\, [d\rho + 3(V_R/R)\, \rho\, dt],$$

$$\Longrightarrow \quad d\rho = -\, 3(V_R/R)\, \rho\, dt.$$

The solution to this equation is:

$$\rho(t) = \rho(t_0)\,\frac{t_0^3}{t^3}\,,\ \text{where } t > t_0\,,$$

or

$$n(t) = n(t_0)\,\frac{t_0^3}{t^3}\,, \text{ where } t > t_0\,. \qquad \text{(IV-1)}$$

Thus the velocity gradient across a spherical shell due to a Hubble-type flow causes the particle density within that flow to decrease as t^{-3} with the restriction that the value taken for t_0 must be after a Hubble-type flow has been established. For my study the values $n(t_0)$ and time t_0 are obtained from published supernova models (see section VI).

V. The Radiative Capture and Annihilation of a Positron

So far I have covered the production and thermalization of positrons. Positrons are destroyed by annihilating with electrons. Positrons can annihilate directly or by forming positronium first. Positronium in the singlet state has a lifetime of 10^{-10} s and emits two 511 keV photons during annihilation. Triplet positronium has a lifetime of 10^{-7}s and undergoes three-photon annihilation (Bussard, Ramaty, and Drachman 1979; Ramaty and Lingenfelter 1987). The probability of annihilation in flight is negligible until the positrons slow to energies of several hundred eV (Bussard, Ramaty, and Drachman 1979). The values of equations (III-4) and (III-9) rise dramatically at low energies (below 0.2 MeV). The positrons will therefore spend only a small amount of time at these energies before thermalizing. I have therefore required all positrons to thermalize before annihilating. At temperatures associated with the interior of a supernova (a few thousand degrees) the dominant mode of annihilation is positronium formation by radiatively recombining with free electrons (Bussard, Ramaty, and Drachman 1979) (see figure 6). The cross section for such a reaction changes with velocity (and therefore temperature). Thankfully the cross section is only proportional to $T^{-1/2}$ (Osterbrock 1974) and is therefore relatively insensitive to these changes.

The theory of radioactive decay is well known. The radiative recombination and subsequent annihilation of a positron can be treated in an analgous manner. The probability that a positron will decay (annihilate) in the next time interval dt is proportional to dt and inversely proportional to the mean lifetime τ :

$$\text{Probability of Decay} = dt/\tau \ . \qquad \text{(V-1)}$$

If the mean lifetime is taken to be a constant, i.e. constant density and constant

recombination rate, and if no new positrons are created then the number of positrons decreases exponentially with time:

$$\frac{dN_+}{N_+} = -\frac{dt}{\tau} ,$$

$$N_+ = N_{+0}\, e^{-\frac{t}{\tau}} . \qquad \text{(V-2)}$$

In an expanding supernova the density is proportional to t^{-3}, as I have shown. Since the mean lifetime against positronium formation is inversely proportional to the density, the mean lifetime goes as t^3 :

$$\tau = \left[n(t_0)\, t_0^3\, Z_{free}\, \lambda \right]^{-1} t^3 = \alpha\, t^3 , \qquad \text{(V-3)}$$

where t_0 = a time when Hubble flow has been established,

$n(t_0)$ = the density of atoms at t_0,

Z_{free} = the number of free electrons per atom at t_0,

λ = the radiative recombination rate,

$$\text{therefore } \frac{dN_+}{N_+} = -\frac{dt}{\alpha t^3} . \qquad \text{(V-4)}$$

There are then two ways of calculating the probability of a positron surviving to a predetermined time:

1) A Monte Carlo simulation using the probability of capture and annihilation,

2) An analytic solution to equation (V-4) which requires taking α as a constant.

The Monte Carlo routine that I used stepped through time with a constant time step. At the beginning of each step the mean lifetime was evaluated for that time. This set the probability of capture for that time step. A random number was generated for each positron that had survived to that time. If the number (between 0 and 1) was less than the probability, the positron was removed. The time was then incremented after all the surviving positrons had been checked for survival. Thus the number of positrons that survived to the "end" time out of the initial number injected could be found. There was one major difficulty with this method. The "resolution" was inversely proportional to the number of positrons initially injected. By "resolution" I mean the smallest probability of survival obtainable. For example, in following the history of 10^4 particles the probability of survival cannot be calculated when it becomes comparable to 10^{-4} or less. The survival fraction is quantized in units of 10^{-4}. In Type II SN the probability of survival is a very small number. To get probabilities down to the size needed would use an enormous amount of computer time. This problem led to the use of the following analytic solution.

The analytic solution I derived from equation (V-4) proved to be more useful for my calculations :

$$\frac{dN_+}{N_+} = -\frac{dt}{\alpha t^3},$$

$$\int_{N_+(t_1)}^{N_+(t_2)} \frac{dN_+}{N_+} = -\frac{1}{\alpha}\int_{t_1}^{t_2} \frac{dt}{t^3},$$

$$\ln\left[\frac{N_+(t_2)}{N_+(t_1)}\right] = \frac{1}{2\alpha}\left[t_2^{-2} - t_1^{-2}\right],$$

$$\frac{N_+(t_2)}{N_+(t_1)} = \exp\left[\frac{1}{2\alpha}\left(t_2^{-2} - t_1^{-2}\right)\right]. \qquad \text{(V-5)}$$

With this formula I could calculate the probability of a positron surviving without using arbitrarily large numbers of initial positrons to get small probabilities.

A comparison of the two methods shows that they produce comparable results when the number of positrons that survive to the end of the monte carlo program is of the order 100 (see Appendix A). Equation (V-5) can be used to produce an analytic model for the survival of positrons but it is difficult to incorporate the thermalization process (see Analytic Models of Positron Survival, section VII).

The advantage of the numerical approach, on the other hand, is that it does not require a constant annihilation rate per free electron. The temperature, ionization, direct in-flight annihilation, etc. can all be represented by time-dependent quantities in a Monte Carlo history. That approach is therefore more general. The temperature of the supernova ejecta should decrease with time due to the fewer number of decays depositing energy within the supernova. The rate of this cooling is unknown because it is uncertain whether a pulsar was created in the SN 1987A. This and other unknowns (degree of radial mixing, expansion velocity, magnetic field configuration, clumping, etc.) coupled with the weak temperature dependence of the recombination rate, λ, led me to believe that my analytic solution was sufficient for this rough survey of positron survival. I have therefore used the approximation that the mean lifetime of positrons against radiative recombination is exactly proportional to t^3 and have esimated the proportionality constants for typical supernova conditions in my models.

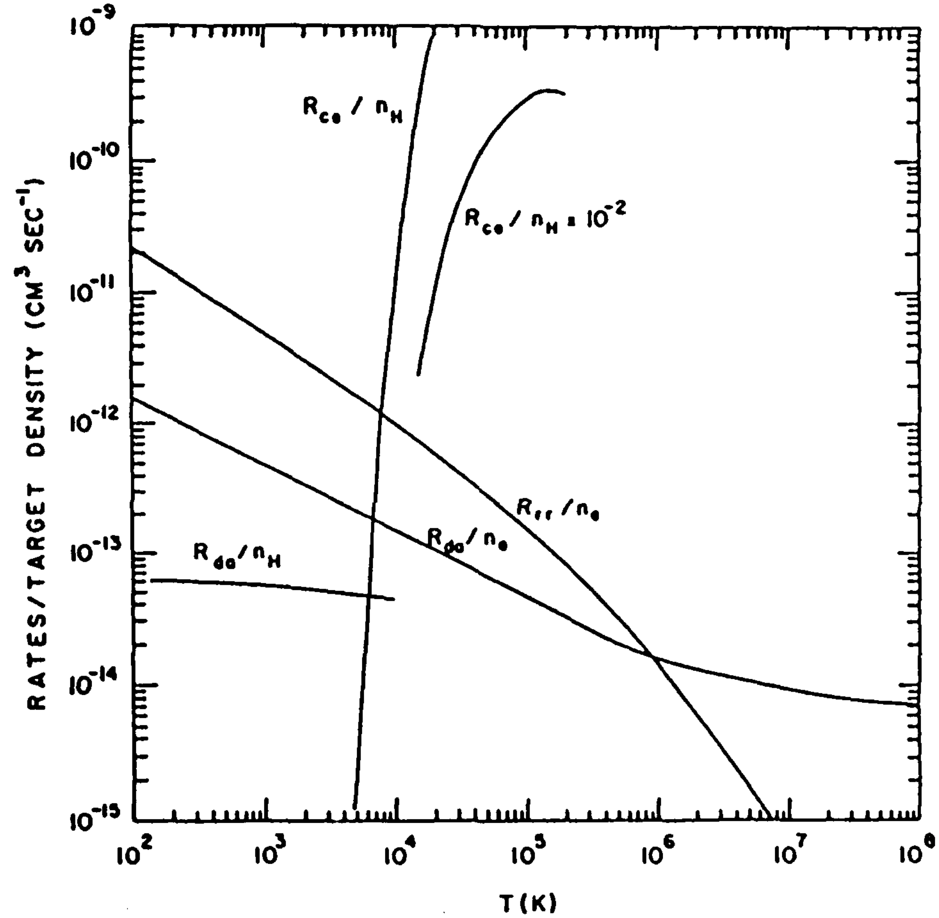


Figure 6: Rates at which thermal positrons for positronium by charge exchange with neutral H, radiative recombination with free electrons, and diect annihilation with both free and bound electrons (Bussard, Ramaty, and Drachman 1979).

VI. Models of SN 1987A

I chose to use Type II supernova models by Woosley et al. and Nomoto et al. as the sources of parameters needed as input in my models. The model of Woosley's that produced the best fit to the light curve of SN 1987A was model 10HM. This model assumed that the progenitor star was Sk -69 202. This star was assumed to have been 20 $M_\odot$ when on the main sequence. Such a star would have a 6 $M_\odot$ helium core. The star was then allowed to lose 4 $M_\odot$ from its hydrogen shell prior to the explosion. A 1.4 $M_\odot$ neutron star was formed from this helium core. The explosion was simulated by replacing the neutron star with a piston that had an energy of 1.45×10^{51} ergs. During the explosion, approximately 0.07 $M_\odot$ of ^{56}Ni was formed in the silicon shell. This model mixed the ^{56}Ni radially outward to a mass coordinate of 9 $M_\odot$. This was done to improve the light curve fit in the 20 - 40 day region. It also accounted for the early detection of x- and γ-rays at 175 days. The regions where the ^{56}Ni was mixed was not homogenized and the model 10H(non-mixed) density profile was left unaltered (Woosley 1988; Woosley (in press); Pinto and Woosley 1988).

Nomoto et al. have also produced several hydrodynamic models of SN 1987A. The model that produced the best fit to the observed light curve was model 11E1. This model also assumed that the progenitor star was 20 $M_\odot$ while on the main sequence. Similarly, a 1.4 $M_\odot$ neutron star was formed from the 6 $M_\odot$ helium core. In this model the star was assumed to have undergone mass loss leaving it with 12.7 $M_\odot$ of material at the time of collapse. The explosion was simulated by replacing the iron core with a point mass and introducing the explosion energy interior to the 11.3 $M_\odot$ of ejecta. Model 11E1 used an explosion energy of 1.00×10^{51} ergs. This model produced appproximately 0.07 $M_\odot$ of ^{56}Ni. Radial mixing was also required in this model to fit observations of the light curve between 25 - 40 days and the early detection of x- and γ-rays. Thus Nomoto mixes the

nickel out to a radial mass coordinate of 10 $M_{\odot}$ (Note: Nomoto and Woosley use different conventions. Nomoto's mass coordinate system has its zero point at the inner edge of the ejecta while Woosley's is at the center of the remnant. Hence forth, I will always add 1.6 $M_{\odot}$ onto Nomoto's coordinate for ease in comparing results.) (Nomoto et al. 1988; Shigeyama, Nomoto, and Hashimoto 1988).

Table VI-1: Summary of supernovae model parameters

	Nomoto et al.	Woosley et al.
Main Sequence Mass	20 $M_{\odot}$	20 $M_{\odot}$
Neutron Star Mass	1.4 $M_{\odot}$	1.4 $M_{\odot}$
Mass of Ejecta	11.3 $M_{\odot}$	14.6 $M_{\odot}$
Energy of Explosion	1.00×10^{51} ergs	1.45×10^{51} ergs
Maximum Radius for Mixing Ni	11.6 $M_{\odot}$ (Woosley Conv.)	9 $M_{\odot}$

The quantities I required from these models were a density profile at an early time after the explosion (but after Hubble flow had been established), a mixing profile, and an approximate temperature of the region. Fortunately all the needed information was available (see figures 7-10). For each model I divided the region of the ejecta which contained nickel into shells. I approximated the density profiles for each model by a series of step functions to obtain a typical density for each shell. I determined an analytic function that fit Woosley's mixing profile and integrated it over the region of each shell to determine the amount of nickel in that shell. Nomoto's mixing was a step function so the amount of nickel in each of those shells was just the area under each step function. A summary of the results follows.

Table VI-2: Summary of model 10HM (Woosley et al.)

Shell ($M_\odot$)	^{56}Ni ($M_\odot$)	$n(t_0)$ (cm)^3	$\rho(t_0)$ (g-cm)^3
1.6 - 2.0	0.0105	1.08×10^{10}	1.0×10^{-12}
2.0 - 2.5	0.0109	5.38×10^{10}	5.0×10^{-12}
2.5 - 3.0	0.0090	3.76×10^{11}	3.5×10^{-11}
3.0 - 3.5	0.0073	5.38×10^{10}	5.0×10^{-12}
3.5 - 4.0	0.0061	3.76×10^{10}	3.5×10^{-12}
4.0 - 4.5	0.0050	2.37×10^{10}	2.2×10^{-12}
4.5 - 5.0	0.0041	2.37×10^{10}	2.2×10^{-12}
5.0 - 5.5	0.0034	2.16×10^{10}	2.0×10^{-12}
5.5 - 6.0	0.0027	1.61×10^{10}	1.5×10^{-12}
6.0 - 6.5	0.0022	3.23×10^{9}	3.0×10^{-13}
6.5 - 7.0	0.0019	2.16×10^{9}	2.0×10^{-13}
7.0 - 7.5	0.0015	2.69×10^{9}	2.5×10^{-13}
7.5 - 8.0	0.0012	3.23×10^{9}	3.0×10^{-13}
8.0 - 8.5	0.0009	3.23×10^{9}	3.0×10^{-13}
8.5 - 9.0	0.0008	3.23×10^{9}	3.0×10^{-13}

(t_0 = 86 days)

Table VI-3: Summary of model 11E1 (Nomoto et al.)

Shell ($M_\odot$)	^{56}Ni ($M_\odot$)	$n(t_0)$ (cm)^3	$\rho(t_0)$ (g-cm)^3
1.6 - 2.1	0.0100	1.08×10^{10}	1.0×10^{-12}
2.1 - 2.6	0.0079	8.60×10^{10}	8.0×10^{-12}
2.6 - 4.0	0.0222	1.08×10^{10}	1.0×10^{-12}
4.0 - 5.1	0.0155	9.68×10^{9}	9.0×10^{-13}
5.1 - 6.2	0.0078	5.38×10^{9}	5.0×10^{-13}
6.2 - 7.6	0.0056	9.68×10^{8}	9.0×10^{-14}
7.6 - 9.6	0.0008	4.30×10^{8}	4.0×10^{-14}
9.6 - 11.6	0.0007	2.15×10^{8}	2.0×10^{-14}

(where t_0 = 116 days)

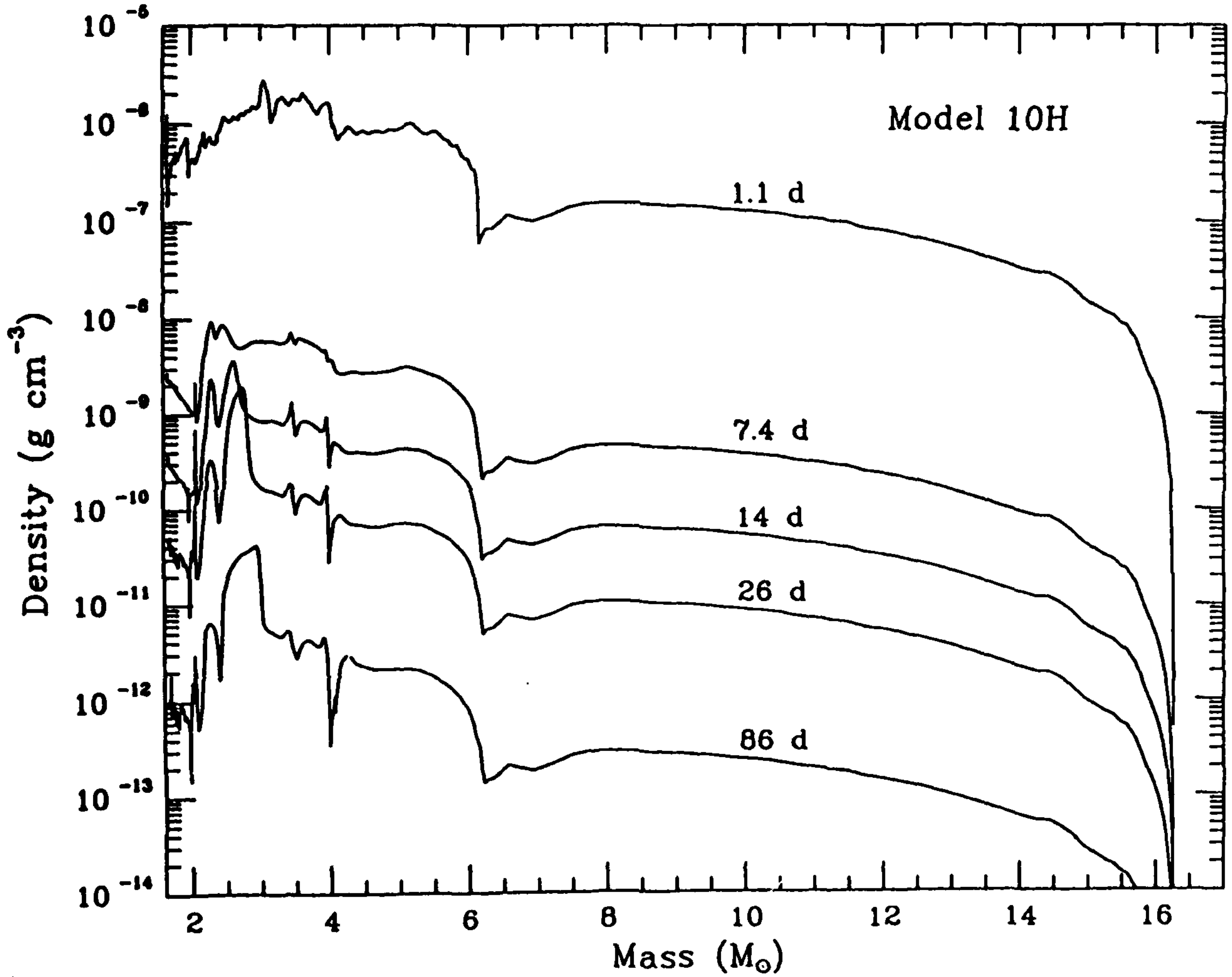


Figure 7: The density profile of Woosley's model 10H at various times after the explosion. I used the profile at 86 days for my initial conditions (Woosley 1988).

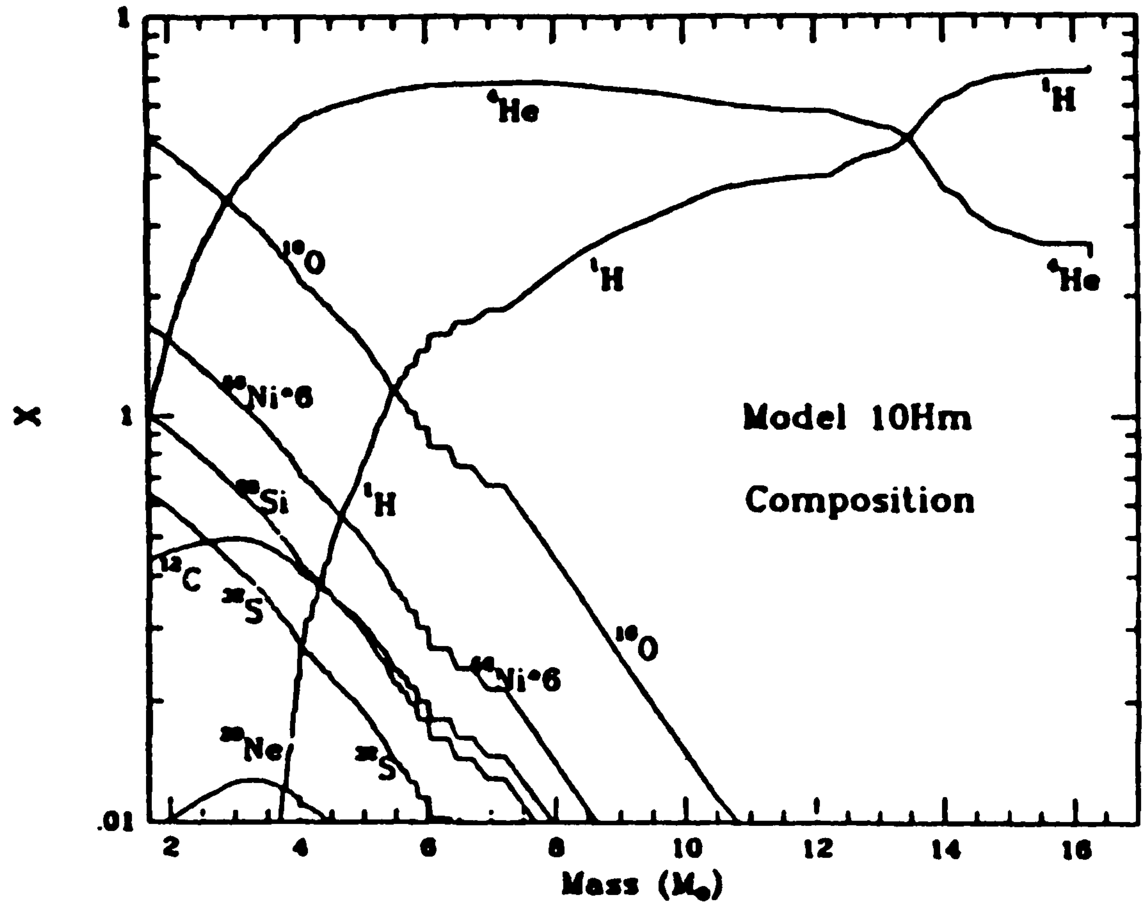


Figure 8: The mixing profile for Woosley's model 10HM. This profile was used to get the amount of cobalt in each shell (Woosley (in press)).

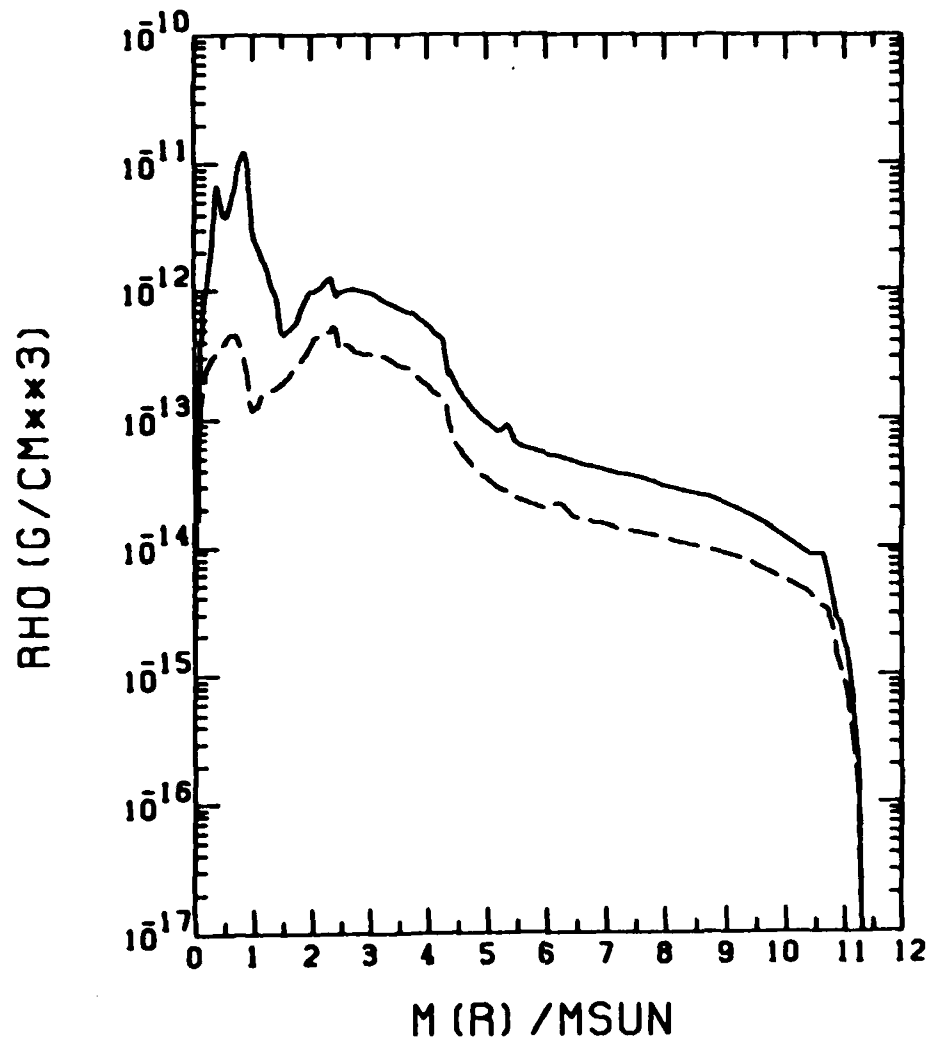


Figure 9: The density profiles for Nomoto's models 11E1 (solid) and 11E2 (dashed) at $t = 116$ days (Shigeyama, Nomoto, Hashimoto 1988).

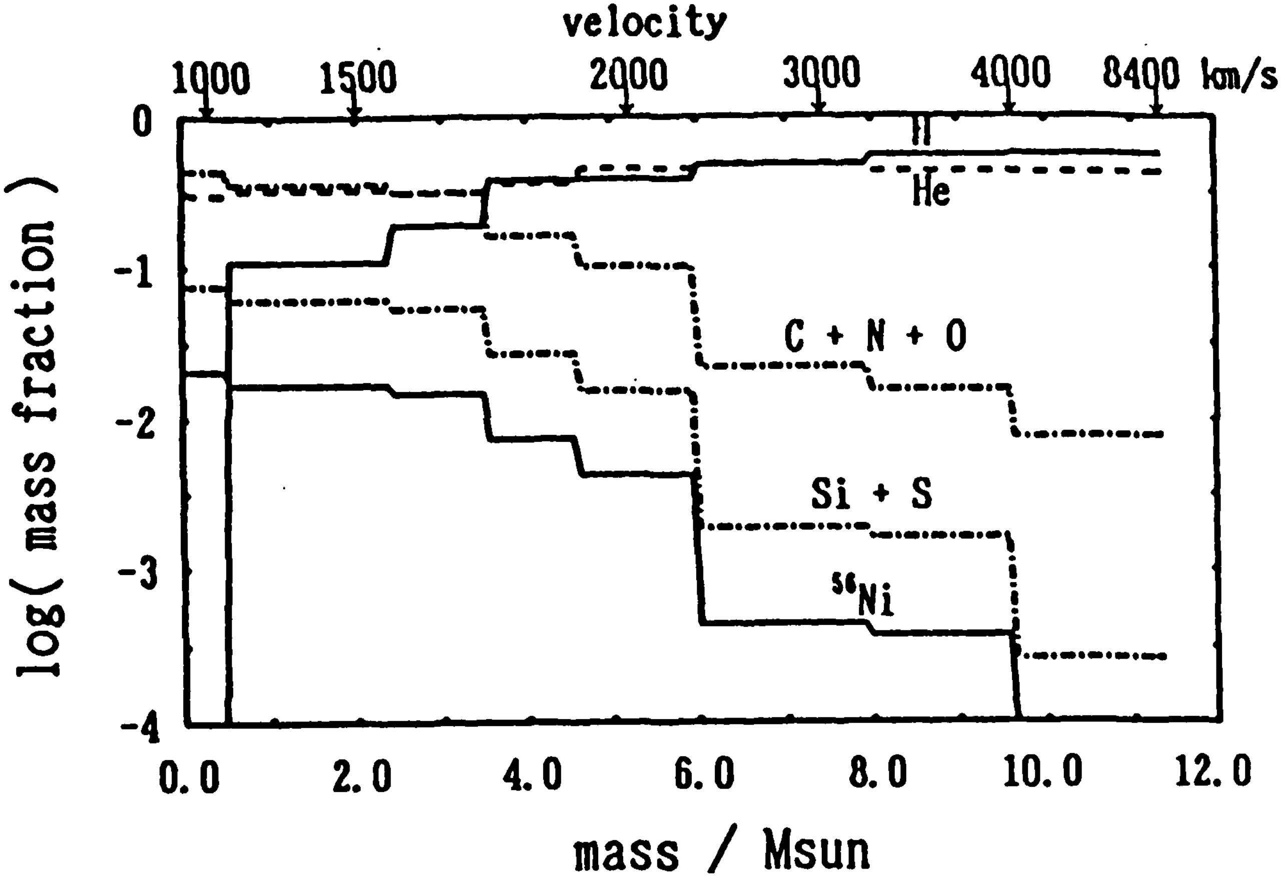


Figure 10: The mixing profile for Nomoto's model 11E1. This profile was used to calculate the amount of cobalt in each shell (Nomoto et al. 1988).

VII. Analytic Models of Positron Survival

I have worked on an analytic model that began with a differential equation involving the time rate of change of the number of positrons per gram. It contained a source term due to the production of positrons by the decay of ^{56}Co and a loss term due to the radiative recombination of positrons with free electrons. The main problem with this solution is that it is hard to incorporate the process of thermalization. But I have found that the time it takes for a positron to become thermal is small compared to time of injection into the supernova (at least for injection times of interest). Therefore the error from this shortcoming should be small.

The analytic solution begins with the expression:

$$\frac{dn_+}{dt} + \lambda\, n_e\, n_+ = S\, e^{-\lambda_0 t}. \qquad \text{(VII-1)}$$

where n_+ = the number of positrons per gram,

S = initial ^{56}Co decay rate per gram = $(N_{Avog.}/56) \times \lambda_0$,

λ_0 = ^{56}Co decay constant = 1/111.52 days,

n_e = the number of free electrons per cm^3 = the number of atoms per cm^3 x Z_{free},

λ = the radiative recombination rate.

But from equation (IV-1) the density of atoms is:

$$n(t) = n(t_0)\,\frac{t_0^3}{t^3}.$$

This changes equation (VII-1) to:

$$\frac{dn_+}{dt} + \left(\lambda\, n(t_0)\, t_0^3\, Z_{free}\right) t^{-3}\, n_+ = S\, e^{-\lambda_0 t} \qquad \text{(VII-2)}$$

An equation of the form $dy/dx+a(x)y = h(x)$ is equivalent to the equation $d(py)/dx=ph$ if :

$$p = \exp\left[\int a(x)\, dx\right].$$

The solution of this equation is (Hildebrand 1976):

$$y = \frac{1}{p}\int p\, h\, dx + \frac{\text{const.}}{p}\ .$$

In particular, if $y = y_1$ at $x = x_1$:

$$y(x) = \int_{x_1}^{x} h(x')\,\frac{p(x')}{p(x)}\,dx' + y_1\,\frac{p(x_1)}{p(x)}. \qquad \text{(VII-3)}$$

By making the substitution in equation (V-3):

$$a(t) = \lambda\, n(t_0)\, t_0^3\, Z_{free}\, t^{-3} = \frac{1}{\alpha t^3}\ ,$$

$$\Rightarrow\ p(t) = \exp\left[-\frac{1}{2\alpha}\left(\frac{1}{t^2}\right)\right].$$

At $t = t_1$, $n_+(t) = n_+(t_1)$ for $t_1 \geq t_0$ (the initial time of the expansion). From equation (VII-3):

$$n_+(t) = n_+(t_1)\exp\left[\frac{1}{2\alpha}\left(\frac{1}{t^2} - \frac{1}{t_1^2}\right)\right] +$$

$$S\exp\left[\frac{1}{2\alpha}\left(\frac{1}{t^2}\right)\right]\int_{t_1}^{t}\exp\left[-\left(\lambda_0\, t' + \frac{1}{2\alpha\, t'^2}\right)\right]dt'. \qquad \text{(VII-4)}$$

The first term corresponds to the number of positrons existing at t_1 that survive until time t. The second term corresponds to the number of positrons emitted between t_1 and t that survive until time t. I then let $t_1 \Rightarrow t_0$ and let $t \Rightarrow \infty$:

$$n_+(\infty) = n_+(t_0)\exp\left[-\frac{1}{2\alpha t_0^2}\right] + S\int_{t_0}^{\infty}\exp\left[-\left(\lambda_0 t' + \frac{1}{2\alpha t'^2}\right)\right]dt'. \qquad \text{(VII-5)}$$

The integral needed to be done numerically. To do this I used a routine on the microVax, DQDAGI, from the IMSL Math Library. When time was kept in seconds it was difficult to get the integrating routine to produce a non-zero solution to the integral in equation VII-5 because of the extremely small size of the integrand over the entire range of the integration except for a very sharp peak at 5×10^8 seconds far from the lower limit of integration. I therefore converted my units of time into years. This moved the peak closer to the lower limit of integration (in the absolute sense) and thus made it harder for the integration routine to ignore. I used this model to calculate the number of positrons that would survive until $t = \infty$ from the innermost shell of Woosley's model 10HM. I used the following values for the input parameters:

$\lambda_0 = 1.038\times10^{-7}\ s^{-1} = 1/111.52$ days $= 3.27\ yr^{-1}$,

$\lambda = 2.00\times10^{-12}\ cm^3\ s^{-1} = 6.31\times10^{-5}$ ($T \approx 3500$ K) (Bussard et al 1979),

$Z_{free} = 1.5$ electrons/atom,

$t_0 = 7.43\times10^6$ s $= 86$ days $= 0.236$ yrs,

$n(t_0) = 1.08\times10^{10}$ atoms cm^{-3},

$\alpha = 7.52\times10^{-20}\ s^{-2} = 7.45\times10^{-5}\ yrs^{-2}$.

Using these parameters, the value for the integral in equation VII-5 was 8.52×10^{-27} s. The first term in equation (VII-5) because it was very small. By taking a value for S of

1.12×10^{15} g^{-1} s^{-1}, I calculated that 9.54×10^{-12} positrons per gram should survive. This translates to 3.99×10^{19} positrons surviving from the inner-most shell of Woosley's model 10HM. This result is compared to that of the computer simulation in section IX.

VIII. Computer Model of Positron Survival

My computer model for positron escape from a Type II supernova drew from the derivations and research presented in sections II - VI of this thesis. Two of the major improvements of this type of model over the previously discussed analytic model were:

1) Requiring positrons to thermalize before annihilating,
2) Allowing the radial mixing of cobalt into regions of lower density.

To accomodate the radial mixing, the supernova ejecta was broken up into a number of shells (as discussed in section VI). There were two sets of initial conditions corresponding to the two supernova models discussed in section VI. Each shell was assigned an initial density and amount of ^{56}Ni (^{56}Co) in accordance with the model being used. A summary of this data was shown in tables VI-2 and VI-3. Other parameters needed in this model were the ^{56}Co decay rate, the radiative recombination rate for positrons and electrons, and the degree of ionization of the ejecta. The values I took for these parameters were the same as I used in the analytic model:

$$\lambda_0 = 1.038\times10^{-7}\ s^{-1},$$
$$\lambda = 2.00\times10^{-12}\ cm^3\ s^{-1},$$
$$Z_{free} = 1.5 \text{ electrons/atom.}$$

This program calculated the number of positrons that survived from each shell. To do this, the continuous emission of positrons was approximated by a discrete set of positron injection times varying from 5 - 30 years with the interval between injection times being 1 year. The number of cobalt decays that occurred within each time interval

dt centered on the injection time can be described by the expression:

$$dN_{Co} = N_{0\,Co}\left[\exp\left(-\lambda_0(t - dt/2)\right) - \exp\left(-\lambda_0(t + dt/2)\right)\right].$$

The number of positrons injected was 1/5 this number.

For each injection time the program calculated a mean time of thermalization for a positron (the time when a typical positron was slowed to thermal energies). This was done by using equations (III-4) and (III-9) to calculate a time of thermalization for a positron in each of the energy intervals listed in table II-1. These times were then weighted by the energy distribution function to get the mean time of thermalization. During the thermalization process the density was allowed to evolve according to equation (IV-1). The program then used equation (V-5) to calculate the fraction of positrons surviving from the mean time of thermalization until $t = \infty$ and that time when the mean lifetime of positrons to radiative recombination, τ, equalled 100 years (as calculated for the central shell of model 10HM). This second calculation was made to estimate the error in terminating the recombination process at approximately 110 years (when $\tau = 100$ years). This could be of use when performing a Monte Carlo history. Thus the number of positrons surviving from a given shell with a given injection time could be calculated by taking the product of the survival fraction and the number of positrons injected for that injection time in that shell. The program stepped through injection times of 5 to 30 years and through all of the shells. The program added the contributions from all times and all shells to get a total number of positrons surviving for each model. A flow diagram for the program along with a printout are included in Appendix B. A quantity of potentially more interest was the probability of a positron surviving from a particular shell. This gives insight into how the survival of positrons depends on the density of a region.

A summary of the results for each shell when the positrons were required to survive

until $t = \infty$ can be seen in tables VIII-1 and VIII-2. Note the large difference from shell to shell in the probability of surviving. This is due to the fact that the probability of surviving is an exponential involving the density and one can see that the density varies significantly from shell to shell. Thus the radial mixing of nickel (cobalt) outward significantly improves the production of positrons able to contribute to a diffuse annihilation line (see figures 11 and 12).

As I have already stated I have also calculated the probability that a positron will survive until the mean lifetime, τ, equalled 100 years (approximately 110 years after the explosion) and the number that will do so from each shell to compare with the previous results (see tables VIII-3 and VIII-4). The differences between the results for survival until $t = \infty$ and until $\tau = 100$ years is small for the shells producing the most positrons. The difference between the overall survival probability and the overall number of "survivors" is correspondingly small, 11.4% for model 10HM and 2.6% for model 11E1 (see table VIII-5).

Table VIII-1: Summary of Results for model 10HM Initial Conditions

Shell ($M_\odot$)	Initial Density (at 86 days)	Probability of Surviving until $t = \infty$	Number That Survive until $t = \infty$
1.6 - 2.0	1.08×10^{10}	1.98×10^{-33}	8.89×10^{19}
2.0 - 2.5	5.38×10^{10}	1.87×10^{-57}	0
2.5 - 3.0	3.76×10^{11}	9.52×10^{-155}	0
3.0 - 3.5	5.38×10^{10}	1.87×10^{-57}	0
3.5 - 4.0	3.76×10^{10}	6.53×10^{-51}	1.71×10^{2}
4.0 - 4.5	2.37×10^{10}	1.40×10^{-43}	3.00×10^{9}
4.5 - 5.0	2.37×10^{10}	1.40×10^{-43}	2.46×10^{9}
5.0 -5.5	2.16×10^{10}	3.08×10^{-42}	4.48×10^{10}
5.5 - 6.0	1.61×10^{10}	2.98×10^{-38}	3.44×10^{14}
6.0 - 6.5	3.23×10^{9}	3.16×10^{-22}	2.97×10^{30}
6.5 - 7.0	2.16×10^{9}	2.14×10^{-19}	1.74×10^{33}
7.0 - 7.5	2.69×10^{9}	6.83×10^{-21}	4.38×10^{31}
7.5 - 8.0	3.23×10^{9}	3.16×10^{-22}	1.62×10^{30}
8.0 - 8.5	3.23×10^{9}	3.16×10^{-22}	1.22×10^{30}
8.5 - 9.0	3.23×10^{9}	3.16×10^{-22}	1.08×10^{30}

Table VIII-2: Summary of Results for model 11E1 Initial Conditions

Shell ($M_\odot$)	Initial Density (at 116 days)	Probability of Surviving until $t = \infty$	Number that Survive until $t = \infty$
1.6 - 2.1	1.08×10^{10}	3.04×10^{-45}	1.30×10^{8}
2.1 - 2.6	8.60×10^{10}	1.80×10^{-105}	0
2.6 - 4.0	1.08×10^{10}	3.04×10^{-45}	2.88×10^{8}
4.0 - 5.1	9.68×10^{9}	1.32×10^{-43}	8.76×10^{9}
5.1 - 6.2	5.38×10^{9}	9.08×10^{-36}	3.03×10^{17}
6.2 - 7.6	9.68×10^{8}	5.01×10^{-20}	1.20×10^{33}
7.6 - 9.6	4.30×10^{8}	3.27×10^{-15}	1.12×10^{37}
9.6 - 11.6	2.15×10^{8}	5.06×10^{-12}	1.51×10^{40}

Table VIII-3: Summary of Results for model 10HM Initial Conditions

Shell ($M_\odot$)	Initial Density (at 86 days)	Probability of Surviving until $\tau = 100$ years	Number That Survive until $\tau = 100$ years
1.6 - 2.0	1.08×10^{10}	3.43×10^{-33}	1.54×10^{20}
2.0 - 2.5	5.38×10^{10}	2.90×10^{-56}	0
2.5 - 3.0	3.76×10^{11}	2.02×10^{-146}	0
3.0 - 3.5	5.38×10^{10}	2.90×10^{-56}	0
3.5 - 4.0	3.76×10^{10}	4.45×10^{-50}	1.16×10^{3}
4.0 - 4.5	2.37×10^{10}	4.70×10^{-43}	1.00×10^{10}
4.5 - 5.0	2.37×10^{10}	4.70×10^{-43}	8.24×10^{9}
5.0 -5.5	2.16×10^{10}	9.27×10^{-42}	1.35×10^{11}
5.5 - 6.0	1.61×10^{10}	6.77×10^{-38}	7.82×10^{14}
6.0 - 6.5	3.23×10^{9}	3.72×10^{-22}	3.50×10^{30}
6.5 - 7.0	2.16×10^{9}	2.39×10^{-19}	1.94×10^{33}
7.0 - 7.5	2.69×10^{9}	7.83×10^{-21}	5.03×10^{31}
7.5 - 8.0	3.23×10^{9}	3.72×10^{-22}	1.91×10^{30}
8.0 - 8.5	3.23×10^{9}	3.72×10^{-22}	1.43×10^{30}
8.5 - 9.0	3.23×10^{9}	3.72×10^{-22}	1.27×10^{30}

Table VIII-4: Summary of Results for model 11E1 Initial Conditions

Shell ($M_\odot$)	Initial Density (at 116 days)	Probability of Surviving until $\tau = 100$ years	Number that Survive until $\tau = 100$ years
1.6 - 2.1	1.08×10^{10}	1.17×10^{-44}	5.01×10^{8}
2.1 - 2.6	8.60×10^{10}	8.47×10^{-101}	0
2.6 - 4.0	1.08×10^{10}	1.17×10^{-44}	1.11×10^{9}
4.0 - 5.1	9.68×10^{9}	4.44×10^{-43}	2.94×10^{10}
5.1 - 6.2	5.38×10^{9}	1.78×10^{-35}	5.94×10^{17}
6.2 - 7.6	9.68×10^{8}	5.66×10^{-20}	1.36×10^{33}
7.6 - 9.6	4.30×10^{8}	3.47×10^{-15}	1.19×10^{37}
9.6 - 11.6	2.15×10^{8}	5.18×10^{-12}	1.56×10^{40}

Table VIII-5: Summary of Overall Results for models 10HM and 11E1

	10HM $t = \infty$	10HM $\tau = 100$	11E1 $t = \infty$	11E1 $\tau = 100$
Probability of Surviving	6.20×10^{-21}	6.93×10^{-21}	5.03×10^{-14}	5.16×10^{-14}
Number of Survivors	1.79×10^{33}	2.00×10^{33}	1.52×10^{40}	1.56×10^{40}
% Difference in number of "survivors"	11.7%		2.6%	

I comment on these results and compare them to the analytic model in section IX.

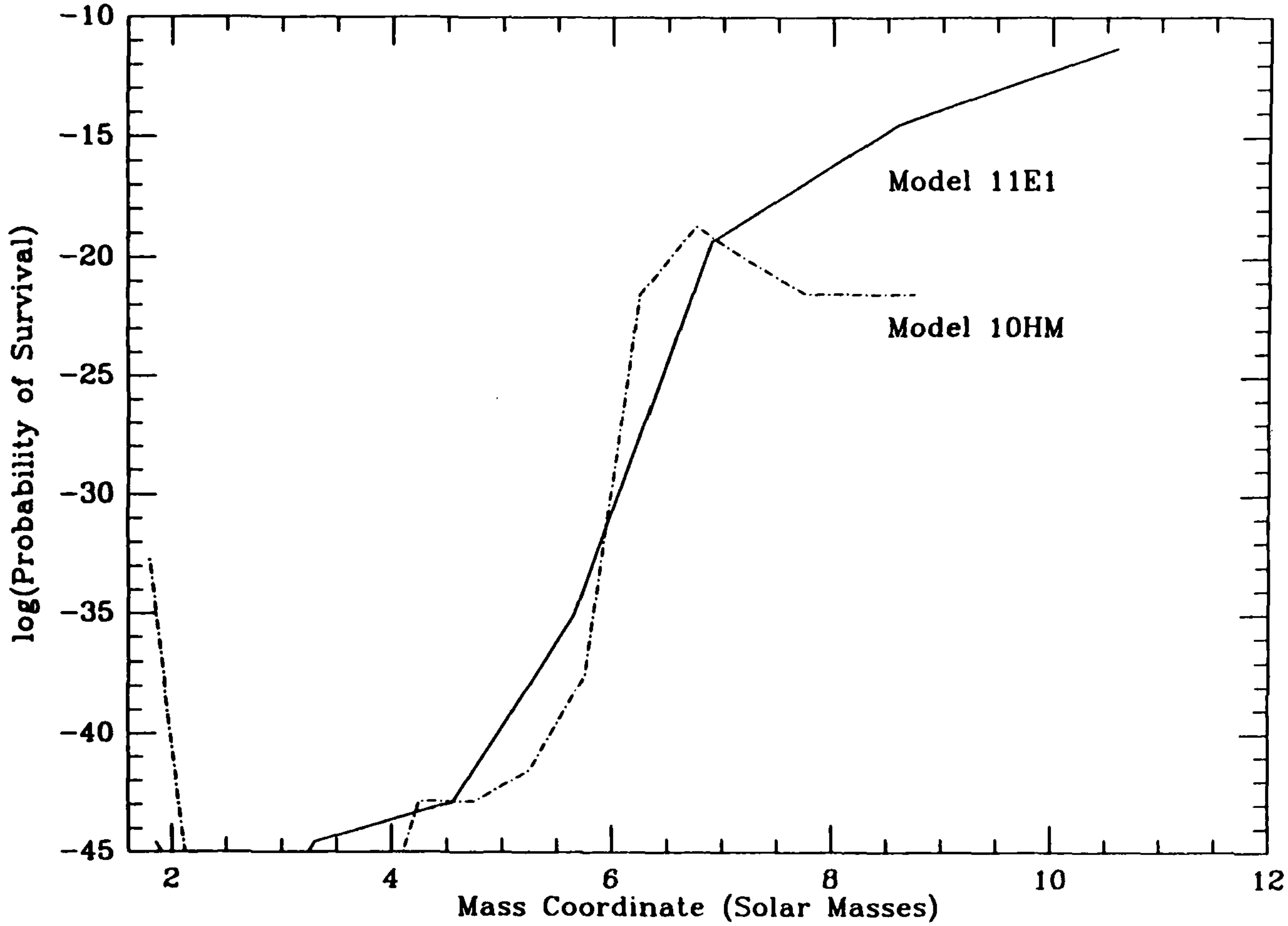


Figure 11: This plot shows the probability of a positron surviving until $t = \infty$ with initial conditions set from each of the models. Note the extremely large range of values due to variations in density.

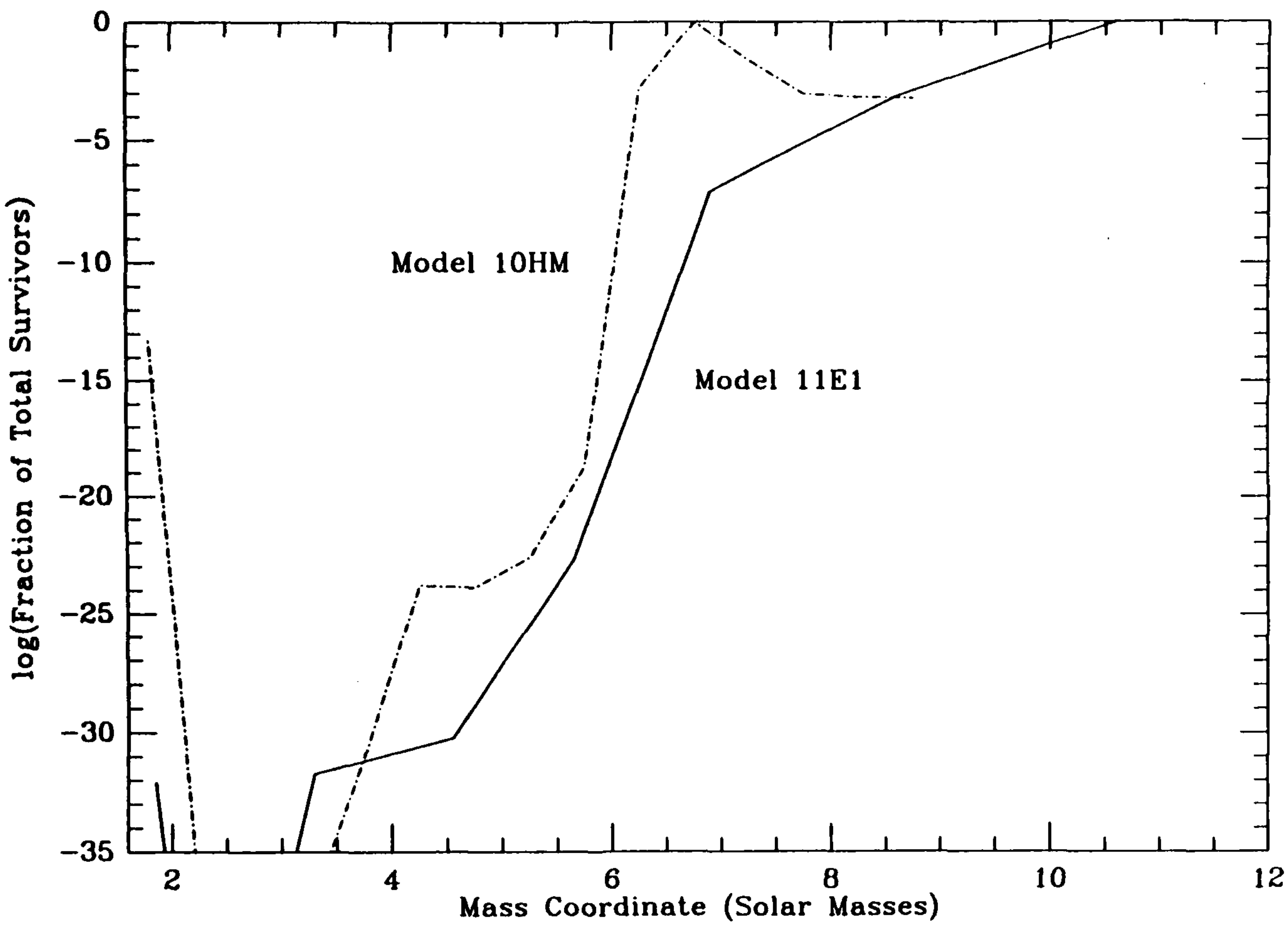


Figure 12: This plot shows fraction of the total number of surviving positrons contributed by each shell for both sets of initial conditions. Note that vast majority of the positrons are coming from the shell with the lowest density.

IX. Summary and Conclusions

My original purpose for undertaking this project was to investigate Type II supernovae as a possible source for the diffuse component of the 511 keV annihilation radiation from the Galactic center. At the recent 14th Texas Symposium on Relativistic Astrophysics, Reuven Ramaty argued that a source of $3x10^{43}$ positrons/sec was needed to account for the diffuse background (this is easily verified to within an order of magnitude by a simple calculation if the distance to the Galactic center is taken to be 8 kpc). My models show that Type II supernovae do not produce nearly enough positrons to account for this line. Using initial conditions set by model 11E1, my models show only $1.5x10^{40}$ positrons will survive (model 10HM inital conditions yielded approximately 10^{33}). If one assumes a fairly optimistic rate of one Type II supernova every 100 years in the Galactic center, a production rate of only $4.777x10^{30}$ positrons/sec is achieved. This is well short of what is needed. This does not mean that this model was a failure. There was good agreement between the analytic model and the computer model. The computer model predicts $8.89x10^{19}$ positrons surviving until $t = \infty$ from the inner shell of model 10HM while the analytic model predicts $3.99x10^{19}$. The diffence can be accounted for by the fact that in the computer model the positrons were required to thermalize before annihilating. During the thermalization process the density in each shell decreased. This coupled with the strong dependence of the probability of survival on the density increases the chance for survival from a particular injection time. As mentioned previously, the integral contained in the analytic solution was difficult to evaluate. A non-zero solution came only after converting seconds to years.

The computer model produced interesting results as I have stated in section VIII. The most interesting is the strong dependence of the probability of survival on the density (see figure 13). The reason for this is that the probability of survival contains an exponential

involving the density. Thus a variation in density from shell to shell of a factor of 50, produces a factor of e^{50} difference in the probability of surviving. This is the reason that mixing has such an overwhelming effect on the number of positrons that survive. Therefore a small refinement in the density profiles of these models to produce a better fit to the light curve would greatly affect the results of my model.

This strong dependence on the density suggests that Type I supernovae with their larger explosion energies (and thus lower densities at a given time) are good candidates for the source of positrons. All that is needed is a nearby Type I supernova to improve theoretical models.

In my computer model I used positron injection times ranging from 5 to 30 years. Figure 14 shows that a peak level of production was reached within that time period for each shell, ie. the injection time that produced the most "survivors" from each shell was between 5 and 30 years. Thus I feel that my models adequately describe the production of Galactic positrons from Type II supernovae given their somewhat limiting assumptions. More complicated hydrodynamic models which take into account the structure of the magnetic fields and the details of the radial mixing may yield more precise results but they should not negate my final result.

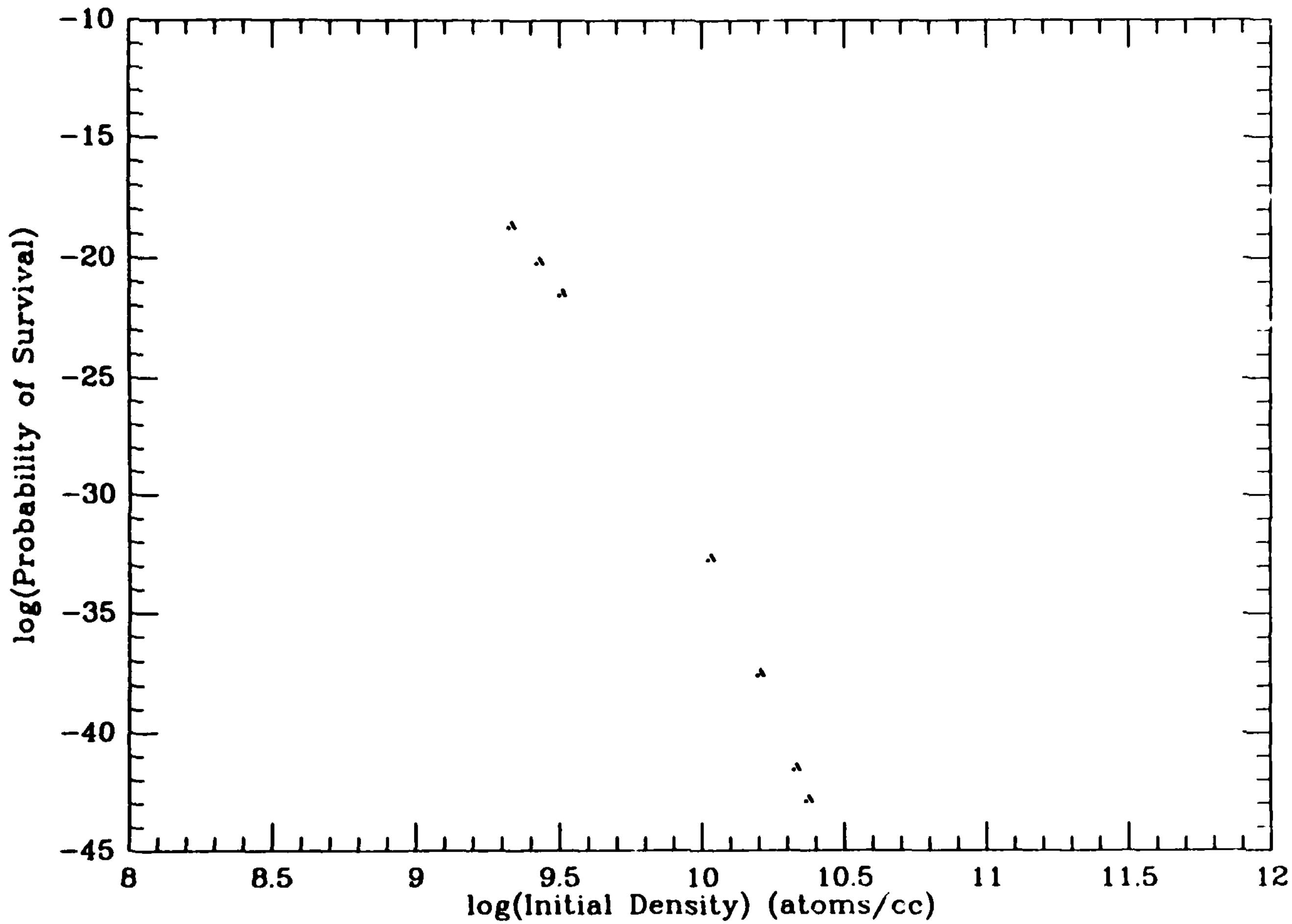


Figure 13: The strong dependences of the probability of survival on the density is evident in this plot. It is interesting to note the almost linear relationship between the logarithm of these two quantities.

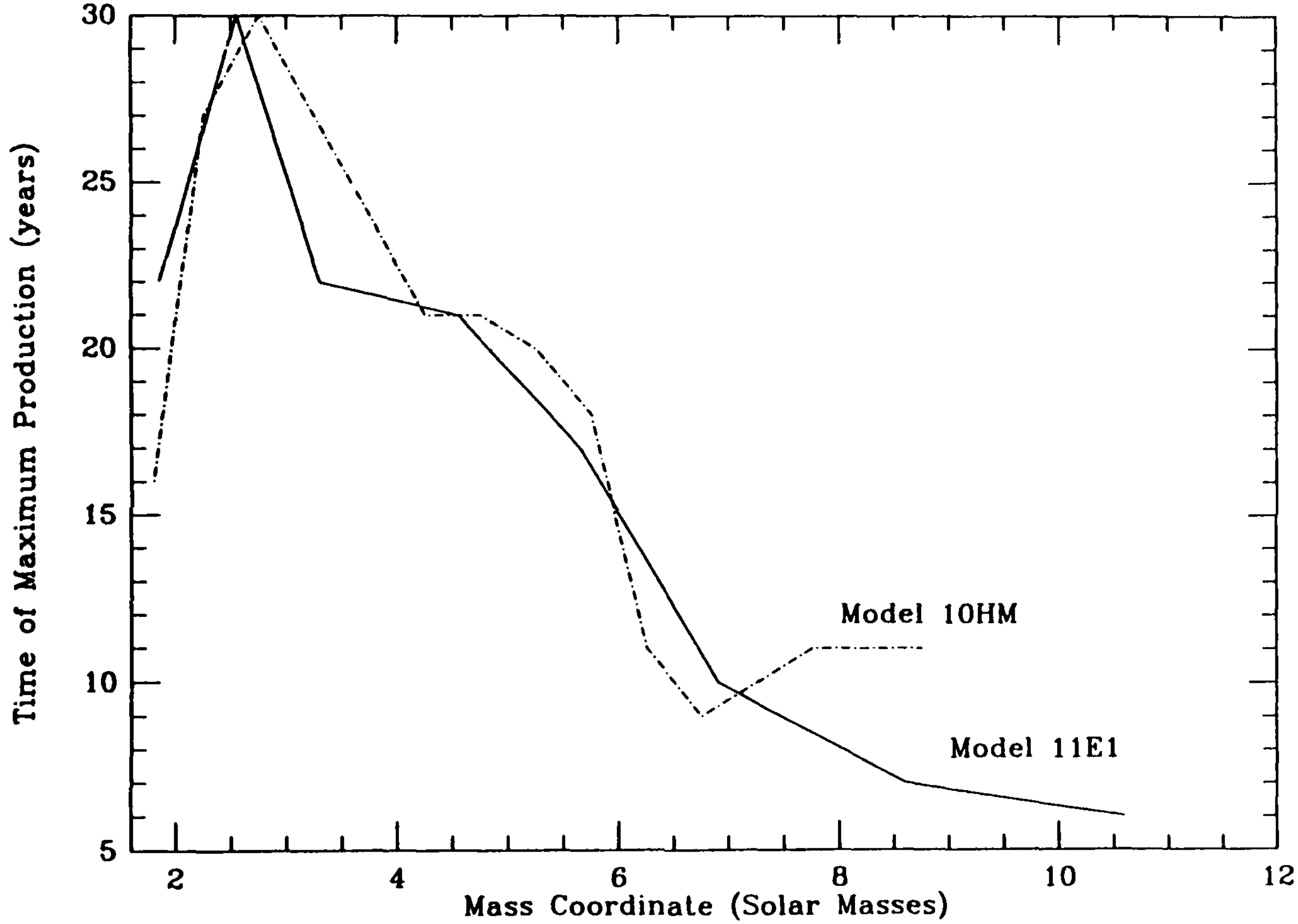


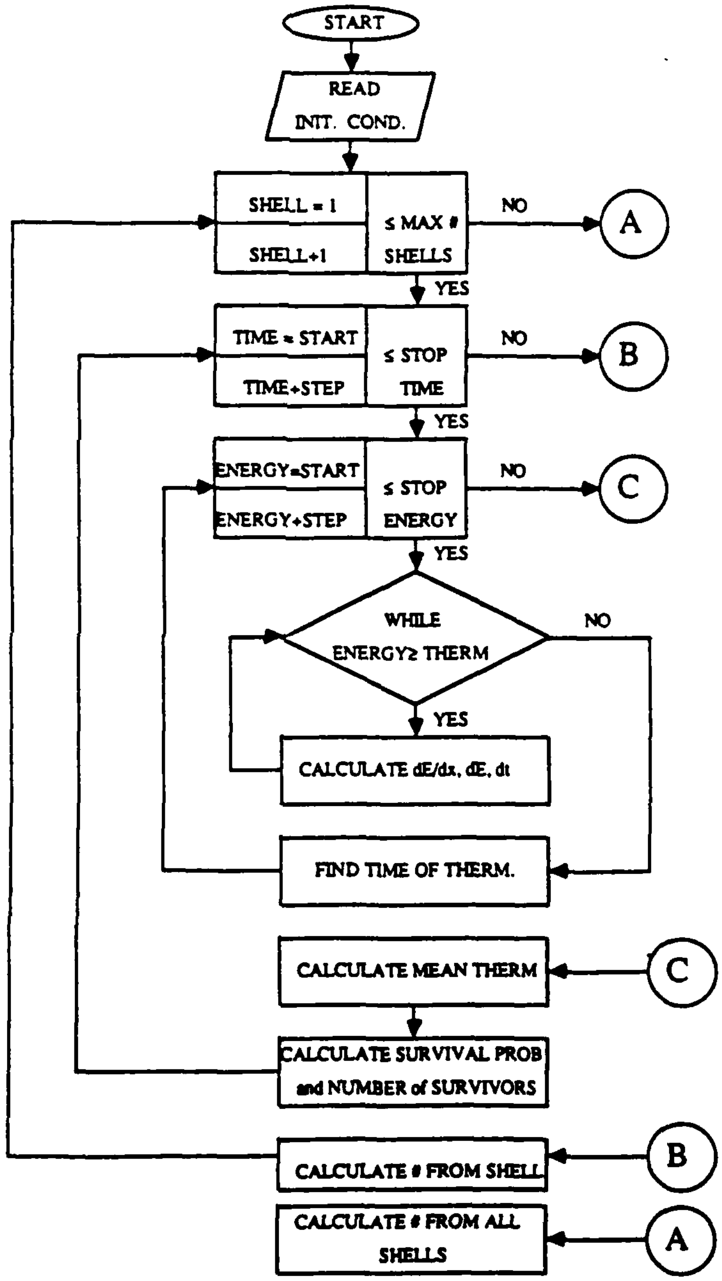
Figure 14: This figure shows the injection time when the largest contribution was made to the "survivors" from that shell. Note that the shells of lowest density made their largest contributions between 6 and 12 years.

Appendix A: Comparison of Analytic and Monte Carlo Survival Routines

Program: fdecay.c Last revision: 1/18/89

Time	Monte Carlo Survival Prob.	Analytic Survival Prob.
9.460800e+08	1.040000e-03	1.035319e-03
9.776160e+08	1.600000e-03	1.658541e-03
1.009152e+09	2.520000e-03	2.543862e-03
1.040688e+09	3.880000e-03	3.755176e-03
1.072224e+09	5.460000e-03	5.358753e-03
1.103760e+09	7.460000e-03	7.420789e-03
1.135296e+09	1.014000e-02	1.000504e-02
1.166832e+09	1.302000e-02	1.317069e-02
1.198368e+09	1.658000e-02	1.697060e-02
1.229904e+09	2.120000e-02	2.144997e-02
1.261440e+09	2.698000e-02	2.664539e-02
1.292976e+09	3.310000e-02	3.258440e-02
1.324512e+09	4.022000e-02	3.928528e-02
1.356048e+09	4.758000e-02	4.675734e-02
1.387584e+09	5.568000e-02	5.500124e-02
1.419120e+09	6.472000e-02	6.400976e-02
1.450656e+09	7.426000e-02	7.376845e-02
1.482192e+09	8.482000e-02	8.425654e-02
1.513728e+09	9.612000e-02	9.544782e-02
1.545264e+09	1.080000e-01	1.073115e-01

Appendix B: Program Flow Chart and Printout



```
/*********************    LIFETIMEW.C    4/19/89    ********************/
#include <stdio.h>
#include <math.h>
 #define CUT_OFF_ENERGY 2.6e-4                          /**in MeV**/
#define INITIAL_DENSITY 1.08e10                         /**atoms/cc, from Woosley's
graph**/
#define INITIAL_TIME 7.43e6                             /**86 days expressed in
seconds**/
#define RAD_RECOM_RATE 2.0e-12                          /**from Bussard's paper**/
#define COBALT_SOURCE 1.4987e54                         /**0.07 solar masses converted to
atoms**/
#define DECAY_RATE 1.03785e-07                          /**1/111.52 days**/
#define ION_POT 2.99e-4                                 /**11.5*Z in Mev**/
#define Z_FREE 1.5                                      /**assuming 1.5 free
electrons/atom**/
#define Z_BOUND 24.5
#define IN_FLIGHT_FRACTION 0.0                          /**Number that annihilate in
flight**/
#define REST_MASS 0.511
#define END_TIME 3.473495e9                             /**when tau is 100 years**/

FILE *point;

/**************************************************************************/
/****************************** SUBROUTINES ******************************/
/**************************************************************************/

/******Ionization Losses using the handbook formula******/

double ionization(energy, density, betasqrd)
double energy, density, betasqrd;
{
      double ratio, part1, part2, part3, dEdx1;
      ratio = pow(energy/REST_MASS, 0.5);
      part1 = 5.1e-25*(density*Z_BOUND)/betasqrd;
      part2 = pow(betasqrd, 0.5)*(energy+REST_MASS)*ratio/ION_POT;
      part3 = 0.5*betasqrd;
      dEdx1 = part1*(log(part2)-part3);
      return (dEdx1);
 }

/**************************************************************************/

/******Plasma Losses using Jackson's formula******/

double plasma(energy, density, betasqrd)
double energy, density, betasqrd;
{
      double gamma, part4, part5, part6, dEdx2;
      gamma = pow((1-betasqrd), -0.5);
      part4 = 5.1e-25*(density*Z_FREE)/betasqrd;
      part5 = 5.3e5*pow(betasqrd, 0.5)*pow((density*Z_FREE), -0.5);
      part6 = 3.86e-11*pow((2.0/(gamma-1.0)), 0.5);
      dEdx2 = part4*log(0.764*part5/part6);
      return (dEdx2);
}

/**************************************************************************/
```

```
/******Radiative Capture of Positrons******/

double decay(meantherm, initdensity)
double meantherm, initdensity;
{
      double fraction, stop, taunot, timesqrd, const;

      timesqrd = pow(meantherm, -2.0);
      stop = pow(END_TIME, -2.0);
      taunot=1/(initdensity*Z_FREE*RAD_RECOM_RATE*pow(INITIAL_TIME, 3.0));
      const = pow((taunot*2), -1.0);
      fraction = exp(const*(stop - timesqrd));

     return(fraction);
}

/****************************************************************************/

/******Number of Positrons Injected at Starttime******/

double injection(starttime, step, shellsource)
double starttime, step, shellsource;
{
     double start, finish, num;

     num=shellsource*(exp(-(DECAY_RATE*(starttime-(step/2)))) -
exp(-(DECAY_RATE*(starttime+(step/2)))));

     return(num);
}

/****************************************************************************/
/****************************** MAIN PROGRAM ******************************/
/****************************************************************************/

main()
{
   double  energy, startEn, time, starttime, step;
   double  density, scale, meantherm;
   double  dEdx, dEdx1, dEdx2, dE, aveenergy;
   double  velocity, dt, total, survivors, numinjected, percentleft;
   double  avebeta, betasqrd;
   double  store[10][2], distrib[10], shell[15][2];
   double  initdensity, shellsource, grandtotal, sum;
   int     x,y,z,s;

   point=fopen("dataW","w");

   fprintf(point,"\n\n");
   fprintf(point,"  PROGRAM: lifetimeW.c    LAST REVISION: 4/19/89\n\n");
   fprintf(point,"  Start: 5 years   Finish: 30 years   Step= 1 year\n\n\n");
   fprintf(point,"----------------------------------------------------------\n\n");

   distrib[0]= 0.061;
   distrib[1]= 0.117;
   distrib[2]= 0.152;
   distrib[3]= 0.165;
   distrib[4]= 0.158;
```

```
distrib[5]= 0.142;
distrib[6]= 0.104;
distrib[7]= 0.066;
distrib[8]= 0.030;
distrib[9]= 0.005;

shell[0][0]= 1.08e10;
shell[1][0]= 5.38e10;
shell[2][0]= 3.76e11;
shell[3][0]= 5.38e10;
shell[4][0]= 3.76e10;
shell[5][0]= 2.37e10;
shell[6][0]= 2.37e10;
shell[7][0]= 2.16e10;
shell[8][0]= 1.61e10;
shell[9][0]= 3.23e9;
shell[10][0]= 2.16e9;
shell[11][0]= 2.69e9;
shell[12][0]= 3.23e9;
shell[13][0]= 3.23e9;
shell[14][0]= 3.23e9;

shell[0][1]= 2.246e53;
shell[1][1]= 2.332e53;
shell[2][1]= 1.925e53;
shell[3][1]= 1.562e53;
shell[4][1]= 1.305e53;
shell[5][1]= 1.070e53;
shell[6][1]= 8.770e52;
shell[7][1]= 7.273e52;
shell[8][1]= 5.773e52;
shell[9][1]= 4.706e52;
shell[10][1]= 4.064e52;
shell[11][1]= 3.209e52;
shell[12][1]= 2.567e52;
shell[13][1]= 1.925e52;
shell[14][1]= 1.711e52;

grandtotal = 0;
step = 3.1536e7;
z = 0;

/***************************START OF SHELL LOOP****************************/

for (z=0; z< 14.5; z=z+1.0)
{
total = 0;
initdensity = shell[z][0];
shellsource = shell[z][1]/5;

fprintf(point," Shell number: %2.0f  Initial density: %5.3e  Amount of cobalt: %5.3e\n\n", z+1.0,
initdensity, shellsource);

fprintf(point," INJECTION TIME   TIME OF THERM.   NUMBER OF POSITRONS   PROB. OF
SURVIVAL\n");
fprintf(point," -----------  -----------  --------------  --------------\n");


/***************************START OF TIME LOOP*****************************/
```

```
for (starttime= 1.5768e8; starttime<= 9.47e8; starttime=starttime + step)
{
 x=0;
 y=0;
 meantherm=0;

/***********************START OF ENERGY LOOP************************/

for (startEn= 0.073; startEn<= 1.46; startEn=startEn + 0.146)
   {
    time=starttime;
    energy=startEn;

   /****************THERMALIZATION LOOP***************************/

   while (energy > CUT_OFF_ENERGY )
     {
      scale=(5.0e10/3.92e21)*pow(time, 3.0)*shell[0][0]/shell[z][0];
      density=initdensity*pow(INITIAL_TIME,3.0)/pow(time,3.0);
      betasqrd=1-pow(((energy/REST_MASS)+1.0), -2.0);
      dEdx1=ionization(energy, density, betasqrd)
      dEdx2=plasma(energy, density, betasqrd);
     dEdx=dEdx1+dEdx2;
     dE=scale*dEdx;
     if(energy-dE>0)
       {
        aveenergy=energy-(dE/2);
         avebeta=pow(1-pow(((aveenergy/REST_MASS)+1.0), -2.0), 0.5);
         velocity=3e10*avebeta;
        dt=scale/velocity;

        /**********Increment Time and Energy**********/

        energy=energy-dE;
        time=time+dt;
       }
      else goto end;
     }

   /***********************END OF WHILE LOOP***********************/

   /**********Calculate and Store Lifetime**********/

   end :;
    store[x++][0]=time;
   }
 /*************************END OF ENERGY LOOP*************************/

/**********Calculate Mean Lifetime**********/

while (y < x)
   {
    store[y][1]=store[y][0]*distrib[y];
    meantherm=meantherm+store[y][1];
    y=y+1;
   }

/**********Calculate How Many Survived**********/
```

```
    percentleft = decay(meantherm, initdensity);
     numinjected = injection(starttime, step, shellsource);
     survivors = percentleft*(numinjected*(1-IN_FLIGHT_FRACTION));
     total = total + survivors;

    fprintf(point," %e     %e      %e        %e\n",starttime,meantherm,numinjected,percentleft);

   }

  /****************************END OF TIME LOOP****************************/

  fprintf(point,"\n\n");     fprintf(point,"Number surviving from shell %1.0f: %e\n\n", z+1.0, total);
  fprintf(point,"------------------------------------------------------------------\n\n");

  grandtotal = grandtotal + total;

  }

  /****************************END OF SHELL LOOP****************************/

  sum=0;

  for(s=0;s<14.5;s=s+1)
  {
     sum=sum+(shell[s][1]/5);
  }

  fprintf(point,"The total number surviving = %e out of %e.\n\n",grandtotal,sum);
   fprintf(point,"The survival fraction is %e.", (grandtotal/sum));

  fclose(point);

}
/*******************************************************************************/
/********************************THE END*******************************************/
/*******************************************************************************/
```

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